Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.6-Cosecant/129-4.6.1.2-d-csc-^n-a+b-csc-^m

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [59]. This is test number [129].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (59)	0.00 (0)
Mathematica	89.83 (53)	10.17 (6)
Fricas	69.49 (41)	30.51 (18)
Maple	69.49 (41)	30.51 (18)
Giac	69.49 (41)	30.51 (18)
Mupad	55.93 (33)	44.07 (26)
Maxima	42.37 (25)	57.63 (34)
Sympy	5.08 (3)	94.92 (56)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

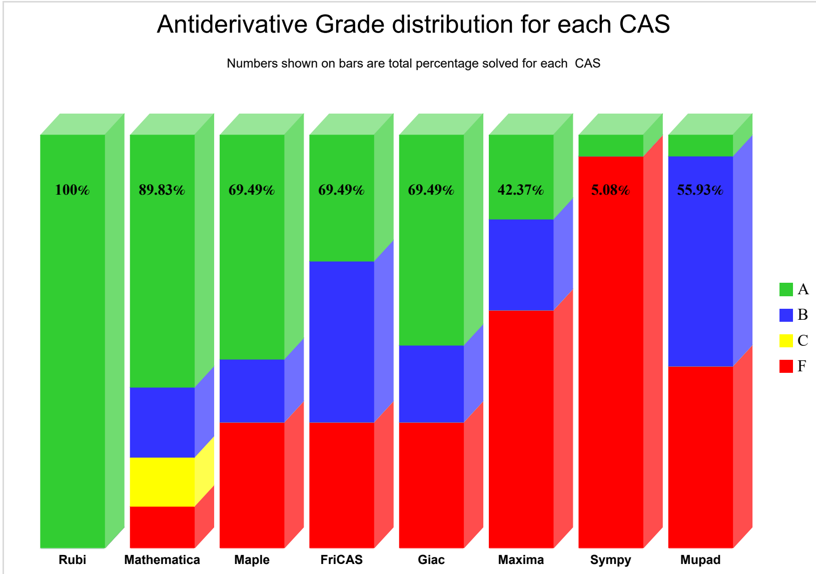
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

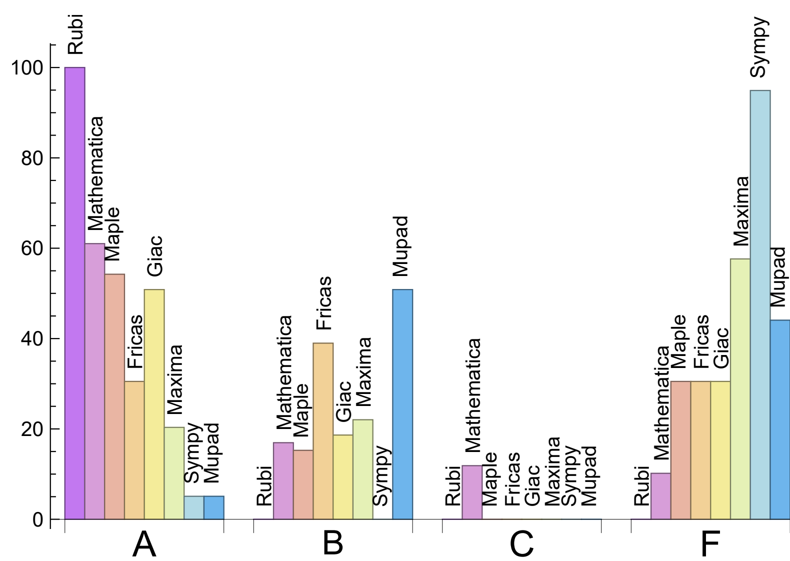
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	61.02	16.95	11.86	10.17
Maple	54.24	15.25	0.00	30.51
Giac	50.85	18.64	0.00	30.51
Fricas	30.51	38.98	0.00	30.51
Maxima	20.34	22.03	0.00	57.63
Mupad	N/A	50.85	0.00	44.07
Sympy	5.08	0.00	0.00	94.92

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	6	100.00 %	0.00 %	0.00 %
Maple	18	100.00 %	0.00 %	0.00 %
Fricas	18	100.00 %	0.00 %	0.00 %
Giac	18	100.00 %	0.00 %	0.00 %
Maxima	34	61.76 %	0.00 %	38.24 %
Sympy	56	94.64 %	1.79 %	3.57 %
Mupad	26	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

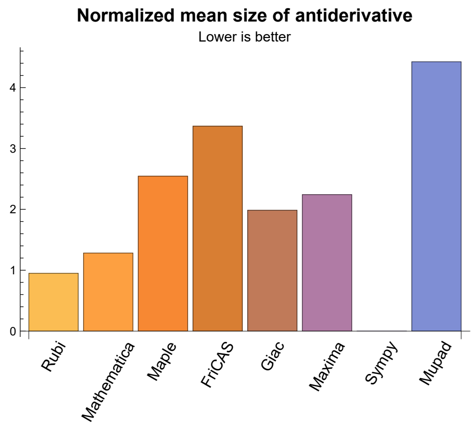
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

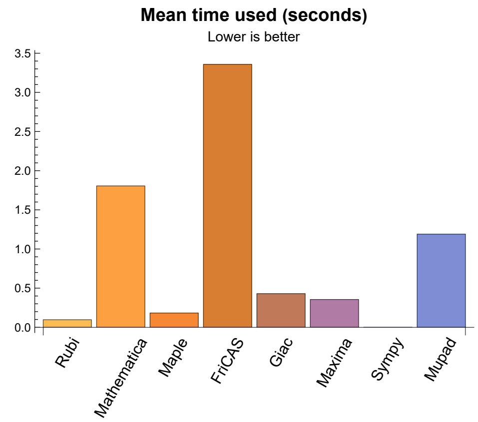
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.10	113.10	0.95	69.00	1.00
Mathematica	1.81	95.26	1.28	76.00	1.16
Maple	0.18	186.07	2.55	77.00	1.32
Maxima	0.36	109.96	2.24	95.00	1.85
Fricas	3.36	253.37	3.37	181.00	3.05
Sympy	0.00	0.00	0.00	0.00	0.00
Giac	0.43	125.54	1.99	91.00	1.58
Mupad	1.19	467.45	4.42	89.00	1.81

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{57, 58, 59\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

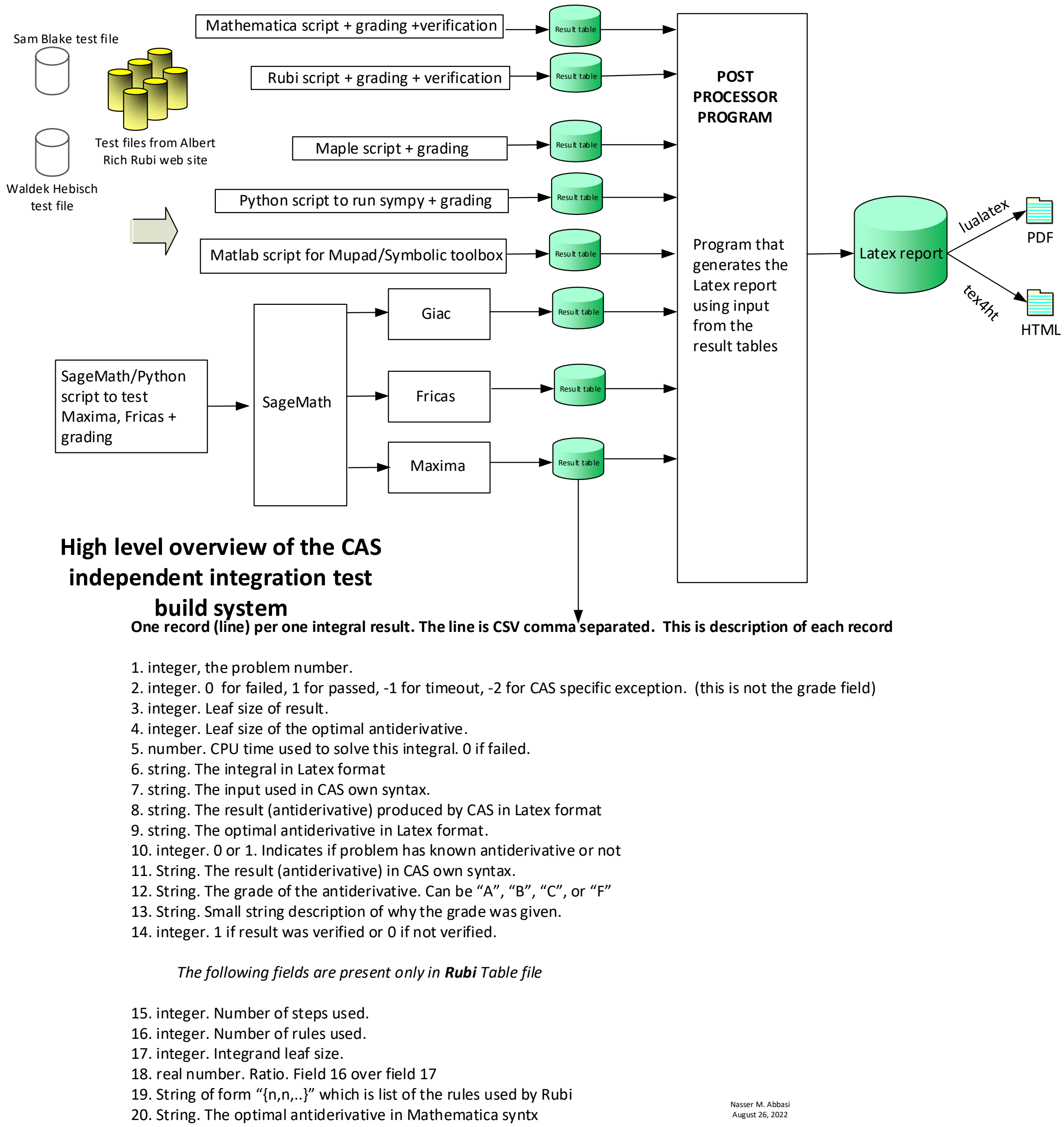
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 28, 29, 30, 31, 32, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 57, 58, 59 }

B grade: { 1, 3, 4, 5, 19, 20, 36, 37, 38, 52 }

C grade: { 21, 22, 23, 24, 25, 26, 27 }

F grade: { 33, 34, 35, 54, 55, 56 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 57, 58, 59 }

B grade: { 13, 14, 15, 16, 17, 18, 19, 20, 51 }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 54, 55, 56 }

2.1.4 Maxima

A grade: { 4, 5, 6, 16, 36, 37, 38, 52, 53, 57, 58, 59 }

B grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17 }

C grade: { }

F grade: { 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56 }

2.1.5 FriCAS

A grade: { 5, 6, 7, 8, 9, 10, 16, 43, 44, 45, 46, 47, 48, 52, 53, 57, 58, 59 }

B grade: { 1, 2, 3, 4, 11, 12, 13, 14, 15, 17, 18, 19, 20, 36, 37, 38, 39, 40, 41, 42, 49, 50, 51 }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 54, 55, 56 }

2.1.6 Sympy

A grade: { 57, 58, 59 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 57, 58, 59 }

B grade: { 13, 14, 15, 16, 17, 18, 19, 20, 36, 38, 51 }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 54, 55, 56 }

2.1.8 Mupad

A grade: { 57, 58, 59 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53 }

C grade: { }

F grade: { 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 54, 55, 56 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	113	68	120	168	0	96	89
N.S.	1	1.00	2.05	1.24	2.18	3.05	0.00	1.75	1.62
time (sec)	N/A	0.050	0.881	0.086	0.258	3.611	0.000	0.426	0.350

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	83	54	97	134	0	73	69
N.S.	1	1.00	1.89	1.23	2.20	3.05	0.00	1.66	1.57
time (sec)	N/A	0.050	0.363	0.067	0.258	2.684	0.000	0.438	0.256

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	63	36	68	91	0	53	49
N.S.	1	1.00	2.33	1.33	2.52	3.37	0.00	1.96	1.81
time (sec)	N/A	0.065	0.171	0.063	0.259	3.331	0.000	0.420	0.253

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	44	21	31	53	0	24	23
N.S.	1	1.00	2.20	1.05	1.55	2.65	0.00	1.20	1.15
time (sec)	N/A	0.041	0.059	0.046	0.262	3.689	0.000	0.427	0.183

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	26	14	16	22	0	13	13
N.S.	1	1.00	2.17	1.17	1.33	1.83	0.00	1.08	1.08
time (sec)	N/A	0.016	0.027	0.033	0.265	4.400	0.000	0.411	0.172

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	47	37	50	54	0	32	27
N.S.	1	1.00	1.68	1.32	1.79	1.93	0.00	1.14	0.96
time (sec)	N/A	0.010	0.092	0.065	0.464	3.546	0.000	0.417	0.216

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	36	78	35	0	44	46
N.S.	1	1.00	1.28	1.44	3.12	1.40	0.00	1.76	1.84
time (sec)	N/A	0.037	0.077	0.063	0.468	3.629	0.000	0.401	0.236

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	58	128	53	0	56	59
N.S.	1	1.00	1.05	1.45	3.20	1.32	0.00	1.40	1.48
time (sec)	N/A	0.047	0.138	0.064	0.534	5.055	0.000	0.422	0.273

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	66	180	70	0	67	78
N.S.	1	1.00	0.92	1.25	3.40	1.32	0.00	1.26	1.47
time (sec)	N/A	0.052	0.181	0.079	0.469	3.438	0.000	0.433	0.347

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	90	230	81	0	91	93
N.S.	1	1.00	0.86	1.36	3.48	1.23	0.00	1.38	1.41
time (sec)	N/A	0.058	0.232	0.084	0.475	3.889	0.000	0.398	0.402

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	108	67	142	124	0	60	52
N.S.	1	1.00	1.89	1.18	2.49	2.18	0.00	1.05	0.91
time (sec)	N/A	0.053	0.342	0.109	0.488	3.007	0.000	0.439	0.406

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	123	97	228	181	0	86	78
N.S.	1	1.00	1.40	1.10	2.59	2.06	0.00	0.98	0.89
time (sec)	N/A	0.084	0.972	0.133	0.474	3.571	0.000	0.434	2.178

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	80	535	417	318	0	250	-1
N.S.	1	1.00	1.23	8.23	6.42	4.89	0.00	3.85	-0.02
time (sec)	N/A	0.068	1.943	0.165	0.480	5.051	0.000	0.584	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	69	275	200	212	0	195	-1
N.S.	1	1.00	1.57	6.25	4.55	4.82	0.00	4.43	-0.02
time (sec)	N/A	0.022	0.098	0.128	0.476	4.140	0.000	0.563	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	32	199	148	120	0	353	-1
N.S.	1	1.00	1.23	7.65	5.69	4.62	0.00	13.58	-0.04
time (sec)	N/A	0.012	0.059	0.223	0.482	3.825	0.000	0.586	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	221	83	219	0	205	-1
N.S.	1	1.00	0.87	3.56	1.34	3.53	0.00	3.31	-0.02
time (sec)	N/A	0.045	0.137	0.125	0.497	2.502	0.000	0.565	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	129	1141	150	427	0	243	-1
N.S.	1	1.00	1.59	14.09	1.85	5.27	0.00	3.00	-0.01
time (sec)	N/A	0.078	0.442	0.132	0.485	4.551	0.000	0.470	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	139	1961	0	546	0	286	-1
N.S.	1	1.00	1.39	19.61	0.00	5.46	0.00	2.86	-0.01
time (sec)	N/A	0.115	0.540	0.116	0.000	3.274	0.000	0.494	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	108	114	0	309	0	111	-1
N.S.	1	1.00	2.92	3.08	0.00	8.35	0.00	3.00	-0.03
time (sec)	N/A	0.041	0.439	1.336	0.000	4.121	0.000	0.806	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	101	117	0	322	0	101	-1
N.S.	1	1.00	2.66	3.08	0.00	8.47	0.00	2.66	-0.03
time (sec)	N/A	0.048	1.349	1.264	0.000	3.846	0.000	0.855	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	102	0	0	0	0	0	-1
N.S.	1	1.00	0.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	7.005	0.157	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	46	0	0	0	0	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.090	1.708	0.121	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	110	0	0	0	0	0	-1
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.123	1.950	0.129	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	120	0	0	0	0	0	-1
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.222	13.574	0.131	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	109	0	0	0	0	0	-1
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.177	7.820	0.124	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	46	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	7.602	0.121	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	72	0	0	0	0	0	-1
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	8.026	0.121	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.682	0.133	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	73	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	1.885	0.142	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	178	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	4.418	0.115	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	126	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	1.225	0.089	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.228	0.088	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.642	0.094	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	3.906	0.113	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	6.819	0.291	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	568	112	125	217	0	205	314
N.S.	1	1.00	5.31	1.05	1.17	2.03	0.00	1.92	2.93
time (sec)	N/A	0.081	6.268	0.169	0.267	3.330	0.000	0.419	0.623

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	152	86	95	155	0	134	234
N.S.	1	1.00	2.08	1.18	1.30	2.12	0.00	1.84	3.21
time (sec)	N/A	0.036	0.688	0.142	0.275	3.477	0.000	0.423	0.417

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	76	46	43	77	0	74	105
N.S.	1	1.00	2.24	1.35	1.26	2.26	0.00	2.18	3.09
time (sec)	N/A	0.019	0.204	0.081	0.285	3.396	0.000	0.415	0.329

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	125	156	0	607	0	194	588
N.S.	1	1.00	1.12	1.39	0.00	5.42	0.00	1.73	5.25
time (sec)	N/A	0.264	1.863	0.205	0.000	3.821	0.000	0.422	0.789

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	144	112	0	524	0	141	515
N.S.	1	1.00	1.71	1.33	0.00	6.24	0.00	1.68	6.13
time (sec)	N/A	0.171	0.535	0.162	0.000	3.263	0.000	0.427	0.580

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	106	77	0	308	0	98	135
N.S.	1	1.00	1.71	1.24	0.00	4.97	0.00	1.58	2.18
time (sec)	N/A	0.103	0.221	0.138	0.000	2.751	0.000	0.417	0.485

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	53	0	245	0	63	129
N.S.	1	1.00	1.17	1.00	0.00	4.62	0.00	1.19	2.43
time (sec)	N/A	0.072	0.068	0.108	0.000	2.790	0.000	0.423	0.429

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	0	154	0	48	36
N.S.	1	1.00	1.00	0.98	0.00	3.85	0.00	1.20	0.90
time (sec)	N/A	0.044	0.029	0.058	0.000	2.944	0.000	0.414	0.287

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	59	68	0	238	0	77	184
N.S.	1	1.00	1.04	1.19	0.00	4.18	0.00	1.35	3.23
time (sec)	N/A	0.042	0.119	0.096	0.000	3.817	0.000	0.426	0.452

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	73	0	235	0	77	766
N.S.	1	1.00	0.92	1.20	0.00	3.85	0.00	1.26	12.56
time (sec)	N/A	0.069	0.104	0.092	0.000	3.157	0.000	0.404	0.652

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	78	112	0	285	0	112	1147
N.S.	1	1.00	0.95	1.37	0.00	3.48	0.00	1.37	13.99
time (sec)	N/A	0.171	0.128	0.103	0.000	3.411	0.000	0.427	0.840

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	145	0	329	0	149	1218
N.S.	1	1.00	0.89	1.32	0.00	2.99	0.00	1.35	11.07
time (sec)	N/A	0.263	0.256	0.125	0.000	4.375	0.000	0.433	0.945

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	129	234	0	410	0	252	1639
N.S.	1	1.00	0.90	1.62	0.00	2.85	0.00	1.75	11.38
time (sec)	N/A	0.376	0.331	0.148	0.000	3.386	0.000	0.408	1.258

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	139	168	0	493	0	158	2677
N.S.	1	1.00	1.29	1.56	0.00	4.56	0.00	1.46	24.79
time (sec)	N/A	0.116	0.460	0.168	0.000	3.687	0.000	0.437	4.179

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	216	314	0	933	0	297	2500
N.S.	1	1.00	1.27	1.85	0.00	5.49	0.00	1.75	14.71
time (sec)	N/A	0.210	1.176	0.259	0.000	3.330	0.000	0.416	8.803

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	279	545	0	1554	0	535	2500
N.S.	1	1.00	1.17	2.28	0.00	6.50	0.00	2.24	10.46
time (sec)	N/A	0.338	2.131	0.389	0.000	4.263	0.000	0.449	12.384

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	66	34	49	33	0	49	39
N.S.	1	1.00	2.13	1.10	1.58	1.06	0.00	1.58	1.26
time (sec)	N/A	0.021	0.053	0.073	0.477	3.439	0.000	0.428	0.245

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	48	71	52	0	45	27
N.S.	1	1.00	0.99	0.71	1.04	0.76	0.00	0.66	0.40
time (sec)	N/A	0.026	0.056	0.071	0.476	3.877	0.000	0.426	0.291

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	4.682	0.115	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.156	2.936	0.095	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	2.124	0.092	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.007	1.700	0.086	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	8.443	0.112	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	6.174	0.309	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [20] had the largest ratio of [28]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	13	0.385
2	A	6	6	1.00	13	0.462
3	A	4	4	1.00	13	0.308
4	A	3	3	1.00	13	0.231
5	A	1	1	1.00	11	0.091
6	A	2	2	1.00	12	0.167
7	A	4	4	1.00	11	0.364
8	A	5	5	1.00	13	0.385
9	A	6	5	1.00	13	0.385
10	A	7	5	1.00	13	0.385
11	A	3	3	1.00	12	0.250
12	A	4	4	1.00	12	0.333
13	A	5	5	1.00	10	0.500
14	A	4	4	1.00	10	0.400
15	A	2	2	1.00	10	0.200
16	A	5	4	1.00	10	0.400
17	A	6	5	1.00	10	0.500
18	A	7	6	1.00	10	0.600
19	A	2	2	1.00	25	0.080
20	A	2	2	1.00	28	0.071
21	A	4	4	1.00	25	0.160
22	A	3	3	1.00	25	0.120
23	A	4	4	1.00	25	0.160
24	A	6	6	1.00	25	0.240
25	A	5	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	6	1.00	25	0.240
27	A	7	6	1.00	25	0.240
28	A	2	2	1.00	23	0.087
29	A	3	3	1.00	24	0.125
30	A	5	5	1.00	21	0.238
31	A	4	4	1.00	21	0.190
32	A	3	3	1.00	19	0.158
33	A	3	3	1.00	12	0.250
34	A	3	3	1.00	19	0.158
35	A	3	3	1.00	21	0.143
36	A	6	5	1.00	12	0.417
37	A	5	4	1.00	12	0.333
38	A	4	4	1.00	12	0.333
39	A	9	9	1.00	13	0.692
40	A	8	8	1.00	13	0.615
41	A	7	7	1.00	13	0.538
42	A	6	6	1.00	13	0.462
43	A	4	4	1.00	11	0.364
44	A	4	4	1.00	12	0.333
45	A	6	6	1.00	11	0.546
46	A	7	7	1.00	13	0.538
47	A	8	7	1.00	13	0.538
48	A	9	7	1.00	13	0.538
49	A	6	6	1.00	12	0.500
50	A	7	7	1.00	12	0.583
51	A	8	7	1.00	12	0.583
52	A	2	2	1.00	12	0.167
53	A	5	4	1.00	12	0.333
54	A	8	5	1.00	21	0.238
55	A	7	4	1.00	21	0.190
56	A	3	3	1.00	19	0.158
57	A	0	0	0.00	0	0.000
58	A	0	0	0.00	0	0.000
59	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{\csc^5(x)}{a+a\csc(x)}\,dx$

Optimal. Leaf size=55

$$\frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{4\cot(x)}{a}-\frac{4\cot^3(x)}{3a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^3(x)}{a+a\csc(x)}$$

```
[Out] 3/2*arctanh(cos(x))/a-4*cot(x)/a-4/3*cot(x)^3/a+3/2*cot(x)*csc(x)/a+cot(x)*
csc(x)^3/(a+a*csc(x))
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3903, 3872, 3853, 3855, 3852}

$$-\frac{4\cot^3(x)}{3a}-\frac{4\cot(x)}{a}+\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{\cot(x)\csc^3(x)}{a\csc(x)+a}+\frac{3\cot(x)\csc(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^5/(a + a*Csc[x]),x]
```

```
[Out] (3*ArcTanh[Cos[x]])/(2*a) - (4*Cot[x])/a - (4*Cot[x]^3)/(3*a) + (3*Cot[x]*C
sc[x])/(2*a) + (Cot[x]*Csc[x]^3)/(a + a*Csc[x])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^5(x)}{a+a\csc(x)}\,dx &= \frac{\cot(x)\csc^3(x)}{a+a\csc(x)}-\frac{\int \csc^3(x)(3a-4a\csc(x))\,dx}{a^2}\\ &= \frac{\cot(x)\csc^3(x)}{a+a\csc(x)}-\frac{3\int \csc^3(x)\,dx}{a}+\frac{4\int \csc^4(x)\,dx}{a}\\ &= \frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^3(x)}{a+a\csc(x)}-\frac{3\int \csc(x)\,dx}{2a}-\frac{4\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cot(x)\right)}{a}\\ &= \frac{3\tanh^{-1}(\cos(x))}{2a}-\frac{4\cot(x)}{a}-\frac{4\cot^3(x)}{3a}+\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^3(x)}{a+a\csc(x)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.
time = 0.88, size = 113, normalized size = 2.05

$$\frac{-20\cot\left(\frac{x}{2}\right)+3\csc^2\left(\frac{x}{2}\right)+36\log\left(\cos\left(\frac{x}{2}\right)\right)-36\log\left(\sin\left(\frac{x}{2}\right)\right)-3\sec^2\left(\frac{x}{2}\right)+8\csc^3(x)\sin^4\left(\frac{x}{2}\right)+\frac{48\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}-\frac{1}{2}\csc^4\left(\frac{x}{2}\right)\sin(x)+20\tan\left(\frac{x}{2}\right)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^5/(a + a*Csc[x]),x]
```

```
[Out] (-20*Cot[x/2] + 3*Csc[x/2]^2 + 36*Log[Cos[x/2]] - 36*Log[Sin[x/2]] - 3*Sec[
x/2]^2 + 8*Csc[x]^3*Sin[x/2]^4 + (48*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (Csc
[x/2]^4*Sin[x])/2 + 20*Tan[x/2])/(24*a)
```

Maple [A]
time = 0.09, size = 68, normalized size = 1.24

method	result	size
default	$\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{x}{2}\right)\right)+7\tan\left(\frac{x}{2}\right)-\frac{1}{3\tan\left(\frac{x}{2}\right)^3}+\frac{1}{\tan\left(\frac{x}{2}\right)^2}-\frac{7}{\tan\left(\frac{x}{2}\right)}-12\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{16}{\tan\left(\frac{x}{2}\right)+1}}{8a}$	68

risch	$-\frac{9i\,e^{5ix}+9\,e^{6ix}-24i\,e^{3ix}-24\,e^{4ix}+7i\,e^{ix}+39\,e^{2ix}-16}{3\left(e^{2ix}-1\right)^{3}\left(i+e^{ix}\right)a}-\frac{3\ln\left(e^{ix}-1\right)}{2a}+\frac{3\ln\left(e^{ix}+1\right)}{2a}$	99
norman	$\frac{-\frac{\tan\left(\frac{x}{2}\right)}{24a}+\frac{\tan^{2}\left(\frac{x}{2}\right)}{12a}-\frac{3\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{4a}+\frac{3\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{4a}-\frac{\tan^{7}\left(\frac{x}{2}\right)}{12a}+\frac{\tan^{8}\left(\frac{x}{2}\right)}{24a}-\frac{15\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4a}}{\tan\left(\frac{x}{2}\right)^{4}\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^5/(a+a*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*(1/3*tan(1/2*x)^3-tan(1/2*x)^2+7*tan(1/2*x)-1/3/tan(1/2*x)^3+1/tan(1/
2*x)^2-7/tan(1/2*x)-12*ln(tan(1/2*x))-16/(tan(1/2*x)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(49) = 98$.

time = 0.26, size = 120, normalized size = 2.18

$$\frac{\frac{21\sin(x)}{\cos(x)+1}-\frac{3\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{\sin(x)^{3}}{(\cos(x)+1)^{3}}}{24\,a}+\frac{\frac{2\sin(x)}{\cos(x)+1}-\frac{18\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{69\sin(x)^{3}}{(\cos(x)+1)^{3}}-1}{24\left(\frac{a\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{a\sin(x)^{4}}{(\cos(x)+1)^{4}}\right)}-\frac{3\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+a*csc(x)),x, algorithm="maxima")
```

```
[Out] 1/24*(21*sin(x)/(cos(x) + 1) - 3*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x)
 + 1)^3)/a + 1/24*(2*sin(x)/(cos(x) + 1) - 18*sin(x)^2/(cos(x) + 1)^2 - 69*
sin(x)^3/(cos(x) + 1)^3 - 1)/(a*sin(x)^3/(cos(x) + 1)^3 + a*sin(x)^4/(cos(x
) + 1)^4) - 3/2*log(sin(x)/(cos(x) + 1))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(49) = 98$.

time = 3.61, size = 168, normalized size = 3.05

$$\frac{32\cos(x)^{4}+14\cos(x)^{3}-48\cos(x)^{2}+9\left(\cos(x)^{4}-2\cos(x)^{2}-\left(\cos(x)^{3}+\cos(x)^{2}-\cos(x)-1\right)\sin(x)+1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-9\left(\cos(x)^{4}-2\cos(x)^{2}-\left(\cos(x)^{3}+\cos(x)^{2}-\cos(x)-1\right)\sin(x)+1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2\left(16\cos(x)^{3}+9\cos(x)^{2}-15\cos(x)-6\right)\sin(x)-18\cos(x)+12}{12\left(a\cos(x)^{4}-2a\cos(x)^{2}-\left(a\cos(x)^{3}+a\cos(x)^{2}-a\cos(x)-a\right)\sin(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+a*csc(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(32*cos(x)^4 + 14*cos(x)^3 - 48*cos(x)^2 + 9*(cos(x)^4 - 2*cos(x)^2 -
(cos(x)^3 + cos(x)^2 - cos(x) - 1)*sin(x) + 1)*log(1/2*cos(x) + 1/2) - 9*(c
os(x)^4 - 2*cos(x)^2 - (cos(x)^3 + cos(x)^2 - cos(x) - 1)*sin(x) + 1)*log(-
1/2*cos(x) + 1/2) + 2*(16*cos(x)^3 + 9*cos(x)^2 - 15*cos(x) - 6)*sin(x) - 1
8*cos(x) + 12)/(a*cos(x)^4 - 2*a*cos(x)^2 - (a*cos(x)^3 + a*cos(x)^2 - a*co
s(x) - a)*sin(x) + a)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^{5}(x)}{\csc(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**5/(a+a*csc(x)),x)
```

```
[Out] Integral(csc(x)**5/(csc(x) + 1), x)/a
```

Giac [A]
time = 0.43, size = 96, normalized size = 1.75

$$-\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,a}+\frac{a^2\tan\left(\frac{1}{2}\,x\right)^3-3\,a^2\tan\left(\frac{1}{2}\,x\right)^2+21\,a^2\tan\left(\frac{1}{2}\,x\right)}{24\,a^3}-\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}+\frac{66\,\tan\left(\frac{1}{2}\,x\right)^3-21\,\tan\left(\frac{1}{2}\,x\right)^2+3\,\tan\left(\frac{1}{2}\,x\right)-1}{24\,a\tan\left(\frac{1}{2}\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+a*csc(x)),x, algorithm="giac")
```

```
[Out] -3/2*log(abs(tan(1/2*x)))/a + 1/24*(a^2*tan(1/2*x)^3 - 3*a^2*tan(1/2*x)^2 +
 21*a^2*tan(1/2*x))/a^3 - 2/(a*(tan(1/2*x) + 1)) + 1/24*(66*tan(1/2*x)^3 -
21*tan(1/2*x)^2 + 3*tan(1/2*x) - 1)/(a*tan(1/2*x)^3)
```

Mupad [B]
time = 0.35, size = 89, normalized size = 1.62

$$\frac{7\tan\left(\frac{x}{2}\right)}{8\,a}-\frac{23\tan\left(\frac{x}{2}\right)^3+6\tan\left(\frac{x}{2}\right)^2-\frac{2\tan\left(\frac{x}{2}\right)}{3}+\frac{1}{3}}{8\,a\tan\left(\frac{x}{2}\right)^4+8\,a\tan\left(\frac{x}{2}\right)^3}-\frac{\tan\left(\frac{x}{2}\right)^2}{8\,a}+\frac{\tan\left(\frac{x}{2}\right)^3}{24\,a}-\frac{3\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^5*(a + a/sin(x))),x)
```

```
[Out] (7*tan(x/2))/(8*a) - (6*tan(x/2)^2 - (2*tan(x/2))/3 + 23*tan(x/2)^3 + 1/3)/
(8*a*tan(x/2)^3 + 8*a*tan(x/2)^4) - tan(x/2)^2/(8*a) + tan(x/2)^3/(24*a) -
(3*log(tan(x/2)))/(2*a)
```

3.2 $\int \frac{\csc^4(x)}{a+a\csc(x)}\,dx$

Optimal. Leaf size=44

$$-\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{2\cot(x)}{a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a+a\csc(x)}$$

```
[Out] -3/2*arctanh(cos(x))/a+2*cot(x)/a-3/2*cot(x)*csc(x)/a+cot(x)*csc(x)^2/(a+a*
csc(x))
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3903, 3872, 3852, 8, 3853, 3855}

$$\frac{2\cot(x)}{a}-\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{\cot(x)\csc^2(x)}{a\csc(x)+a}-\frac{3\cot(x)\csc(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Csc[x]),x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/(2*a) + (2*Cot[x])/a - (3*Cot[x]*Csc[x])/(2*a) + (Cot[
x]*Csc[x]^2)/(a + a*Csc[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(x)}{a+a\csc(x)}\,dx &= \frac{\cot(x)\csc^2(x)}{a+a\csc(x)}-\frac{\int \csc^2(x)(2a-3a\csc(x))\,dx}{a^2}\\
&= \frac{\cot(x)\csc^2(x)}{a+a\csc(x)}-\frac{2\int \csc^2(x)\,dx}{a}+\frac{3\int \csc^3(x)\,dx}{a}\\
&= -\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a+a\csc(x)}+\frac{3\int \csc(x)\,dx}{2a}+\frac{2\text{Subst}(\int 1\,dx,x,\cot(x))}{a}\\
&= -\frac{3\tanh^{-1}(\cos(x))}{2a}+\frac{2\cot(x)}{a}-\frac{3\cot(x)\csc(x)}{2a}+\frac{\cot(x)\csc^2(x)}{a+a\csc(x)}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 83, normalized size = 1.89

$$\frac{4\cot\left(\frac{x}{2}\right)-\csc^2\left(\frac{x}{2}\right)-12\log\left(\cos\left(\frac{x}{2}\right)\right)+12\log\left(\sin\left(\frac{x}{2}\right)\right)+\sec^2\left(\frac{x}{2}\right)-\frac{16\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}-4\tan\left(\frac{x}{2}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Csc[x]),x]
[Out] (4*Cot[x/2] - Csc[x/2]^2 - 12*Log[Cos[x/2]] + 12*Log[Sin[x/2]] + Sec[x/2]^2
 - (16*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - 4*Tan[x/2])/(8*a)
```

Maple [A]
time = 0.07, size = 54, normalized size = 1.23

method	result	size

default	$\frac{\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-2\tan\left(\frac{x}{2}\right)+\frac{8}{\tan\left(\frac{x}{2}\right)+1}-\frac{1}{2\tan\left(\frac{x}{2}\right)^2}+\frac{2}{\tan\left(\frac{x}{2}\right)}+6\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4a}$	54
norman	$\frac{\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{\tan\left(\frac{x}{2}\right)}{8a}+\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{8a}-\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{8a}+\frac{\tan^6\left(\frac{x}{2}\right)}{8a}}{\tan\left(\frac{x}{2}\right)^3\left(\tan\left(\frac{x}{2}\right)+1\right)}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$	81
risch	$\frac{-5\,\mathrm{e}^{2ix}+3i\mathrm{e}^{3ix}+3\,\mathrm{e}^{4ix}+4-i\mathrm{e}^{ix}}{\left(\mathrm{e}^{2ix}-1\right)^2\left(i+\mathrm{e}^{ix}\right)a}+\frac{3\ln\left(\mathrm{e}^{ix}-1\right)}{2a}-\frac{3\ln\left(\mathrm{e}^{ix}+1\right)}{2a}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*(1/2*tan(1/2*x)^2-2*tan(1/2*x)+8/(tan(1/2*x)+1)-1/2/tan(1/2*x)^2+2/ta
n(1/2*x)+6*ln(tan(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(40) = 80$.
time = 0.26, size = 97, normalized size = 2.20

$$-\frac{\frac{4\sin(x)}{\cos(x)+1}-\frac{\sin(x)^2}{(\cos(x)+1)^2}}{8a}+\frac{\frac{3\sin(x)}{\cos(x)+1}+\frac{20\sin(x)^2}{(\cos(x)+1)^2}-1}{8\left(\frac{a\sin(x)^2}{(\cos(x)+1)^2}+\frac{a\sin(x)^3}{(\cos(x)+1)^3}\right)}+\frac{3\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*csc(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*sin(x)/(cos(x) + 1) - sin(x)^2/(cos(x) + 1)^2)/a + 1/8*(3*sin(x)/(c
os(x) + 1) + 20*sin(x)^2/(cos(x) + 1)^2 - 1)/(a*sin(x)^2/(cos(x) + 1)^2 + a
*sin(x)^3/(cos(x) + 1)^3) + 3/2*log(sin(x)/(cos(x) + 1))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(40) = 80$.
time = 2.68, size = 134, normalized size = 3.05

$$\frac{8\cos(x)^3+6\cos(x)^2-3\left(\cos(x)^3+\cos(x)^2+\left(\cos(x)^2-1\right)\sin(x)-\cos(x)-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^3+\cos(x)^2+\left(\cos(x)^2-1\right)\sin(x)-\cos(x)-1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\left(4\cos(x)^2+\cos(x)-2\right)\sin(x)-6\cos(x)-4}{4\left(a\cos(x)^3+a\cos(x)^2-a\cos(x)+\left(a\cos(x)^2-a\right)\sin(x)-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*csc(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*cos(x)^3 + 6*cos(x)^2 - 3*(cos(x)^3 + cos(x)^2 + (cos(x)^2 - 1)*sin(
x) - cos(x) - 1)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^3 + cos(x)^2 + (cos(x)^2
 - 1)*sin(x) - cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2*(4*cos(x)^2 + cos(x)
- 2)*sin(x) - 6*cos(x) - 4)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) + (a*cos(x)
^2 - a)*sin(x) - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(x)}{\csc(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*csc(x)),x)
```

```
[Out] Integral(csc(x)**4/(csc(x) + 1), x)/a
```

Giac [A]
time = 0.44, size = 73, normalized size = 1.66

$$\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,a} + \frac{a\tan\left(\frac{1}{2}\,x\right)^2 - 4\,a\tan\left(\frac{1}{2}\,x\right)}{8\,a^2} + \frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)} - \frac{18\,\tan\left(\frac{1}{2}\,x\right)^2 - 4\,\tan\left(\frac{1}{2}\,x\right)+1}{8\,a\tan\left(\frac{1}{2}\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*csc(x)),x, algorithm="giac")
```

```
[Out] 3/2*log(abs(tan(1/2*x)))/a + 1/8*(a*tan(1/2*x)^2 - 4*a*tan(1/2*x))/a^2 + 2/
(a*(tan(1/2*x) + 1)) - 1/8*(18*tan(1/2*x)^2 - 4*tan(1/2*x) + 1)/(a*tan(1/2*
x)^2)
```

Mupad [B]
time = 0.26, size = 69, normalized size = 1.57

$$\frac{10\tan\left(\frac{x}{2}\right)^2 + \frac{3\tan\left(\frac{x}{2}\right)}{2} - \frac{1}{2}}{4\,a\tan\left(\frac{x}{2}\right)^3 + 4\,a\tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)}{2\,a} + \frac{\tan\left(\frac{x}{2}\right)^2}{8\,a} + \frac{3\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + a/sin(x))),x)
```

```
[Out] ((3*tan(x/2))/2 + 10*tan(x/2)^2 - 1/2)/(4*a*tan(x/2)^2 + 4*a*tan(x/2)^3) -
tan(x/2)/(2*a) + tan(x/2)^2/(8*a) + (3*log(tan(x/2)))/(2*a)
```

3.3 $\int \frac{\csc^3(x)}{a+a\csc(x)}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}(\cos(x))}{a}-\frac{\cot(x)}{a}-\frac{\cot(x)}{a+a\csc(x)}$$

```
[Out] arctanh(cos(x))/a-cot(x)/a-cot(x)/(a+a*csc(x))
```

Rubi [A]
time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.308$, Rules used = {3875, 3874, 3855, 3879}

$$-\frac{\cot(x)}{a}+\frac{\tanh^{-1}(\cos(x))}{a}-\frac{\cot(x)}{a\csc(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Csc[x]),x]
```

```
[Out] ArcTanh[Cos[x]]/a - Cot[x]/a - Cot[x]/(a + a*Csc[x])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(x)}{a+a\csc(x)}\,dx &= -\frac{\cot(x)}{a} - \int \frac{\csc^2(x)}{a+a\csc(x)}\,dx \\ &= -\frac{\cot(x)}{a} - \frac{\int \csc(x)\,dx}{a} + \int \frac{\csc(x)}{a+a\csc(x)}\,dx \\ &= \frac{\tanh^{-1}(\cos(x))}{a} - \frac{\cot(x)}{a} - \frac{\cot(x)}{a+a\csc(x)}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 63 vs. $2(27) = 54$.
time = 0.17, size = 63, normalized size = 2.33

$$\frac{-\cot\left(\frac{x}{2}\right) + 2\log\left(\cos\left(\frac{x}{2}\right)\right) - 2\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{4\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)} + \tan\left(\frac{x}{2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Csc[x]),x]
```

```
[Out] (-Cot[x/2] + 2*Log[Cos[x/2]] - 2*Log[Sin[x/2]] + (4*Sin[x/2])/(Cos[x/2] + S
in[x/2]) + Tan[x/2])/(2*a)
```

Maple [**A**]
time = 0.06, size = 36, normalized size = 1.33

method	result	size
default	$\frac{\tan\left(\frac{x}{2}\right)-\frac{4}{\tan\left(\frac{x}{2}\right)+1}-\frac{1}{\tan\left(\frac{x}{2}\right)}-2\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$	36
norman	$\frac{-\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}-\frac{\tan\left(\frac{x}{2}\right)}{2a}+\frac{\tan^4\left(\frac{x}{2}\right)}{2a}}{\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$	59
risch	$-\frac{2\left(e^{2ix}-2+ie^{ix}\right)}{\left(e^{2ix}-1\right)\left(i+e^{ix}\right)a}+\frac{\ln\left(e^{ix}+1\right)}{a}-\frac{\ln\left(e^{ix}-1\right)}{a}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(tan(1/2*x)-4/(tan(1/2*x)+1)-1/tan(1/2*x)-2*ln(tan(1/2*x)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(27) = 54$.
time = 0.26, size = 68, normalized size = 2.52

$$-\frac{\frac{5\sin(x)}{\cos(x)+1}+1}{2\left(\frac{a\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)} - \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a} + \frac{\sin(x)}{2a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*csc(x)),x, algorithm="maxima")
```

```
[Out] -1/2*(5*sin(x)/(cos(x) + 1) + 1)/(a*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x
) + 1)^2) - log(sin(x)/(cos(x) + 1))/a + 1/2*sin(x)/(a*(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(27) = 54$.
time = 3.33, size = 91, normalized size = 3.37

$$\frac{4\cos(x)^2 + (\cos(x)^2 - (\cos(x)+1)\sin(x) - 1)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) - (\cos(x)^2 - (\cos(x)+1)\sin(x) - 1)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 2(2\cos(x)+1)\sin(x) + 2\cos(x) - 2}{2\left(a\cos(x)^2 - (a\cos(x)+a)\sin(x) - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*csc(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cos(x)^2 + (cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*log(1/2*cos(x) + 1/2
) - (cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*log(-1/2*cos(x) + 1/2) + 2*(2*cos(
x) + 1)*sin(x) + 2*cos(x) - 2)/(a*cos(x)^2 - (a*cos(x) + a)*sin(x) - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(x)}{\csc(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*csc(x)),x)
```

```
[Out] Integral(csc(x)**3/(csc(x) + 1), x)/a
```

Giac [A]
time = 0.42, size = 53, normalized size = 1.96

$$-\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a} + \frac{\tan\left(\frac{1}{2}x\right)}{2a} + \frac{\tan\left(\frac{1}{2}x\right)^2 - 4\tan\left(\frac{1}{2}x\right) - 1}{2\left(\tan\left(\frac{1}{2}x\right)^2 + \tan\left(\frac{1}{2}x\right)\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*csc(x)),x, algorithm="giac")
```

```
[Out] -log(abs(tan(1/2*x)))/a + 1/2*tan(1/2*x)/a + 1/2*(tan(1/2*x)^2 - 4*tan(1/2*
x) - 1)/((tan(1/2*x)^2 + tan(1/2*x))*a)
```

Mupad [B]
time = 0.25, size = 49, normalized size = 1.81

$$\frac{\tan\left(\frac{x}{2}\right)}{2a} - \frac{5\tan\left(\frac{x}{2}\right) + 1}{2a\tan\left(\frac{x}{2}\right)^2 + 2a\tan\left(\frac{x}{2}\right)} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + a/sin(x))),x)
```

```
[Out] tan(x/2)/(2*a) - (5*tan(x/2) + 1)/(2*a*tan(x/2) + 2*a*tan(x/2)^2) - log(tan
(x/2))/a
```

3.4 $\int \frac{\csc^2(x)}{a+a\csc(x)}\,dx$

Optimal. Leaf size=20

$$-\frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cot(x)}{a+a\csc(x)}$$

```
[Out] -arctanh(cos(x))/a+cot(x)/(a+a*csc(x))
```

Rubi [A]
time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {3874, 3855, 3879}

$$\frac{\cot(x)}{a\csc(x)+a}-\frac{\tanh^{-1}(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + a*Csc[x]),x]
```

```
[Out] -(ArcTanh[Cos[x]]/a) + Cot[x]/(a + a*Csc[x])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a+a\csc(x)}\,dx &= \frac{\int \csc(x)\,dx}{a}-\int \frac{\csc(x)}{a+a\csc(x)}\,dx\\ &= -\frac{\tanh^{-1}(\cos(x))}{a}+\frac{\cot(x)}{a+a\csc(x)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(20) = 40.
time = 0.06, size = 44, normalized size = 2.20

$$\frac{-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Csc[x]),x]
[Out] (-Log[Cos[x/2]] + Log[Sin[x/2]] - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2]))/a
```

Maple [A]
time = 0.05, size = 21, normalized size = 1.05

method	result	size
default	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)+\frac{2}{\tan\left(\frac{x}{2}\right)+1}}{a}$	21
norman	$\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$	24
risch	$\frac{2}{(i+e^{ix})a}+\frac{\ln\left(e^{ix}-1\right)}{a}-\frac{\ln\left(e^{ix}+1\right)}{a}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*csc(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(ln(tan(1/2*x))+2/(tan(1/2*x)+1))
```

Maxima [A]
time = 0.26, size = 31, normalized size = 1.55

$$\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}+\frac{2}{a+\frac{a\sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*csc(x)),x, algorithm="maxima")
[Out] log(sin(x)/(cos(x) + 1))/a + 2/(a + a*sin(x)/(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(20) = 40.
time = 3.69, size = 53, normalized size = 2.65

$$-\frac{(\cos(x)+\sin(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+\sin(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\cos(x)+2\sin(x)-2}{2\left(a\cos(x)+a\sin(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*csc(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(x) + sin(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + sin(x) + 1)*l
og(-1/2*cos(x) + 1/2) - 2*cos(x) + 2*sin(x) - 2)/(a*cos(x) + a*sin(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(x)}{\csc(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*csc(x)),x)
```

```
[Out] Integral(csc(x)**2/(csc(x) + 1), x)/a
```

Giac [A]
time = 0.43, size = 24, normalized size = 1.20

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a} + \frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*csc(x)),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/a + 2/(a*(tan(1/2*x) + 1))
```

Mupad [B]
time = 0.18, size = 23, normalized size = 1.15

$$\frac{2}{a\,\left(\tan\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + a/sin(x))),x)
```

```
[Out] 2/(a*(tan(x/2) + 1)) + log(tan(x/2))/a
```

3.5 $\int \frac{\csc(x)}{a+a\csc(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\cot(x)}{a+a\csc(x)}$$

```
[Out] -cot(x)/(a+a*csc(x))
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3879}

$$-\frac{\cot(x)}{a\csc(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Csc[x]),x]
[Out] -(Cot[x]/(a + a*Csc[x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc(x)}{a+a\csc(x)}\,dx = -\frac{\cot(x)}{a+a\csc(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.
time = 0.03, size = 26, normalized size = 2.17

$$\frac{2\sin\left(\frac{x}{2}\right)}{a\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Csc[x]),x]
[Out] (2*Sin[x/2])/(a*(Cos[x/2] + Sin[x/2]))
```

Maple [A]
time = 0.03, size = 14, normalized size = 1.17

method	result	size
default	$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$	14
norman	$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}$	14
risch	$-\frac{2}{\left(i+e^{i x}\right) a}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/(tan(1/2*x)+1)
```

Maxima [A]
time = 0.27, size = 16, normalized size = 1.33

$$-\frac{2}{a+\frac{a\sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*csc(x)),x, algorithm="maxima")
```

```
[Out] -2/(a + a*sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 4.40, size = 22, normalized size = 1.83

$$-\frac{\cos\left(x\right)-\sin\left(x\right)+1}{a\cos\left(x\right)+a\sin\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*csc(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc\left(x\right)}{\csc\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*csc(x)),x)
```

```
[Out] Integral(csc(x)/(csc(x) + 1), x)/a
```

Giac [A]
time = 0.41, size = 13, normalized size = 1.08

$$-\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*csc(x)),x, algorithm="giac")
```

```
[Out] -2/(a*(tan(1/2*x) + 1))
```

Mupad [B]
time = 0.17, size = 13, normalized size = 1.08

$$-\frac{2}{a\,\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + a/sin(x))),x)
```

```
[Out] -2/(a*(tan(x/2) + 1))
```

3.6 $\int \frac{1}{a+a\csc(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{x}{a}+\frac{\cot(c+dx)}{d(a+a\csc(c+dx))}$$

```
[Out] x/a+cot(d*x+c)/d/(a+a*csc(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3862, 8}

$$\frac{\cot(c+dx)}{d(a\csc(c+dx)+a)}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Csc[c + d*x])^(-1),x]
```

```
[Out] x/a + Cot[c + d*x]/(d*(a + a*Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\csc(c+dx)}\,dx &= \frac{\cot(c+dx)}{d(a+a\csc(c+dx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}+\frac{\cot(c+dx)}{d(a+a\csc(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 47, normalized size = 1.68

$$\frac{c+dx-\frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Csc[c + d*x])^(-1),x]
```

```
[Out] (c + d*x - (2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(a*d
)
```

Maple [A]
time = 0.06, size = 37, normalized size = 1.32

method	result	size
risch	$\frac{x}{a}+\frac{2}{da\left(i+e^{i(dx+c)}\right)}$	29
derivativedivides	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}}{ad}$	37
default	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}}{ad}$	37
norman	$\frac{\frac{x}{a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*csc(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*(1/2*arctan(tan(1/2*d*x+1/2*c))+1/2/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.46, size = 50, normalized size = 1.79

$$\frac{2\left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 1/(a + a*sin(d*x + c)/(cos(d
*x + c) + 1)))/d
```

Fricas [A]
time = 3.55, size = 54, normalized size = 1.93

$$\frac{dx+(dx+1)\cos(dx+c)+(dx-1)\sin(dx+c)+1}{ad\cos(dx+c)+ad\sin(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x + (d*x + 1)*cos(d*x + c) + (d*x - 1)*sin(d*x + c) + 1)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\csc(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c)),x)
```

```
[Out] Integral(1/(csc(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.42, size = 32, normalized size = 1.14

$$\frac{\frac{dx+c}{a}+\frac{2}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a + 2/(a*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

Mupad [B]
time = 0.22, size = 27, normalized size = 0.96

$$\frac{x}{a}+\frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/sin(c + d*x)),x)
```

```
[Out] x/a + 2/(a*d*(tan(c/2 + (d*x)/2) + 1))
```

3.7 $\int \frac{\sin(x)}{a+a\csc(x)}\,dx$

Optimal. Leaf size=25

$$-\frac{x}{a}-\frac{2\cos(x)}{a}+\frac{\cos(x)}{a+a\csc(x)}$$

```
[Out] -x/a-2*cos(x)/a+cos(x)/(a+a*csc(x))
```

Rubi [A]
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3904, 3872, 2718, 8}

$$-\frac{x}{a}-\frac{2\cos(x)}{a}+\frac{\cos(x)}{a\csc(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + a*Csc[x]),x]
```

```
[Out] -(x/a) - (2*Cos[x])/a + Cos[x]/(a + a*Csc[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin(x)}{a+a\csc(x)}\,dx = \frac{\cos(x)}{a+a\csc(x)} - \frac{\int(-2a+a\csc(x))\sin(x)\,dx}{a^2}$$
$$= \frac{\cos(x)}{a+a\csc(x)} - \frac{\int 1\,dx}{a} + \frac{2\int\sin(x)\,dx}{a}$$
$$= -\frac{x}{a} - \frac{2\cos(x)}{a} + \frac{\cos(x)}{a+a\csc(x)}$$

Mathematica [A]
time = 0.08, size = 32, normalized size = 1.28

$$-\frac{x+\cos(x)-\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Csc[x]),x]
[Out] -((x + Cos[x] - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2]))/a)
```

Maple [A]
time = 0.06, size = 36, normalized size = 1.44

method	result	size
default	$\frac{-\frac{2}{\tan^2\left(\frac{x}{2}\right)+1}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\frac{2}{\tan\left(\frac{x}{2}\right)+1}}{a}$	36
risch	$-\frac{x}{a}-\frac{e^{ix}}{2a}-\frac{e^{-ix}}{2a}-\frac{2}{(i+e^{ix})a}$	43
norman	$\frac{-\frac{4}{a}-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}-\frac{x}{a}-\frac{x\tan\left(\frac{x}{2}\right)}{a}-\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}-\frac{x\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)\left(\tan\left(\frac{x}{2}\right)+1\right)}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*csc(x)),x,method=_RETURNVERBOSE)
[Out] 8/a*(-1/4/(tan(1/2*x)^2+1)-1/4*arctan(tan(1/2*x))-1/4/(tan(1/2*x)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(25) = 50.
time = 0.47, size = 78, normalized size = 3.12

$$-\frac{2\left(\frac{\sin(x)}{\cos(x)+1}+\frac{\sin(x)^2}{(\cos(x)+1)^2}+2\right)}{a+\frac{a\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}+\frac{a\sin(x)^3}{(\cos(x)+1)^3}}-\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*csc(x)),x, algorithm="maxima")
```

```
[Out] -2*(sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 + 2)/(a + a*sin(x)/(cos(x
) + 1) + a*sin(x)^2/(cos(x) + 1)^2 + a*sin(x)^3/(cos(x) + 1)^3) - 2*arctan(
sin(x)/(cos(x) + 1))/a
```

Fricas [A]
time = 3.63, size = 35, normalized size = 1.40

$$-\frac{(x+2)\cos(x)+\cos(x)^2+(x+\cos(x)-1)\sin(x)+x+1}{a\cos(x)+a\sin(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*csc(x)),x, algorithm="fricas")
```

```
[Out] -((x + 2)*cos(x) + cos(x)^2 + (x + cos(x) - 1)*sin(x) + x + 1)/(a*cos(x) +
a*sin(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(x)}{\csc(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*csc(x)),x)
```

```
[Out] Integral(sin(x)/(csc(x) + 1), x)/a
```

Giac [A]
time = 0.40, size = 44, normalized size = 1.76

$$-\frac{x}{a}-\frac{2\left(\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)+2\right)}{\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)^2+\tan\left(\frac{1}{2}x\right)+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*csc(x)),x, algorithm="giac")
```

```
[Out] -x/a - 2*(tan(1/2*x)^2 + tan(1/2*x) + 2)/((tan(1/2*x)^3 + tan(1/2*x)^2 + ta
n(1/2*x) + 1)*a)
```

Mupad [B]
time = 0.24, size = 46, normalized size = 1.84

$$-\frac{x}{a}-\frac{2\tan\left(\frac{x}{2}\right)^2+2\tan\left(\frac{x}{2}\right)+4}{a\left(\tan\left(\frac{x}{2}\right)^2+1\right)\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + a/sin(x)),x)
[Out] - x/a - (2*tan(x/2) + 2*tan(x/2)^2 + 4)/(a*(tan(x/2)^2 + 1)*(tan(x/2) + 1))
```

3.8 $\int \frac{\sin^2(x)}{a+a\csc(x)}\,dx$

Optimal. Leaf size=40

$$\frac{3x}{2a}+\frac{2\cos(x)}{a}-\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin(x)}{a+a\csc(x)}$$

```
[Out] 3/2*x/a+2*cos(x)/a-3/2*cos(x)*sin(x)/a+cos(x)*sin(x)/(a+a*csc(x))
```

Rubi [A]
time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3904, 3872, 2715, 8, 2718}

$$\frac{3x}{2a}+\frac{2\cos(x)}{a}-\frac{3\sin(x)\cos(x)}{2a}+\frac{\sin(x)\cos(x)}{a\csc(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + a*Csc[x]),x]
```

```
[Out] (3*x)/(2*a) + (2*Cos[x])/a - (3*Cos[x]*Sin[x])/(2*a) + (Cos[x]*Sin[x])/(a + 
 a*Csc[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{a+a\csc(x)}\,dx &= \frac{\cos(x)\sin(x)}{a+a\csc(x)}-\frac{\int(-3a+2a\csc(x))\sin^2(x)\,dx}{a^2}\\ &= \frac{\cos(x)\sin(x)}{a+a\csc(x)}-\frac{2\int\sin(x)\,dx}{a}+\frac{3\int\sin^2(x)\,dx}{a}\\ &= \frac{2\cos(x)}{a}-\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin(x)}{a+a\csc(x)}+\frac{3\int 1\,dx}{2a}\\ &= \frac{3x}{2a}+\frac{2\cos(x)}{a}-\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin(x)}{a+a\csc(x)}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 42, normalized size = 1.05

$$-\frac{-6x-4\cos(x)+\frac{8\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}+\sin(2x)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Csc[x]),x]
[Out] -1/4*(-6*x - 4*Cos[x] + (8*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + Sin[2*x])/a
```

Maple [A]
time = 0.06, size = 58, normalized size = 1.45

method	result
risch	$\frac{3x}{2a}+\frac{e^{ix}}{2a}+\frac{e^{-ix}}{2a}+\frac{2}{(i+e^{ix})a}-\frac{\sin(2x)}{4a}$
default	$\frac{\frac{2\left(\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}+\tan^2\left(\frac{x}{2}\right)-\frac{\tan\left(\frac{x}{2}\right)}{2}+1\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+3\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\frac{16}{8\tan\left(\frac{x}{2}\right)+8}}{a}$
norman	$\frac{\frac{3}{a}-\frac{\tan^5\left(\frac{x}{2}\right)}{a}+\frac{2\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}+\frac{\tan^3\left(\frac{x}{2}\right)}{a}+\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}+\frac{3x}{2a}+\frac{3x\tan\left(\frac{x}{2}\right)}{2a}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2a}+\frac{3x\left(\tan^5\left(\frac{x}{2}\right)\right)}{2a}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2\left(\tan\left(\frac{x}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/a*(1/8*(1/2*tan(1/2*x)^3+tan(1/2*x)^2-1/2*tan(1/2*x)+1)/(tan(1/2*x)^2+1)
^2+3/16*arctan(tan(1/2*x))+1/8/(tan(1/2*x)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(36) = 72$.
time = 0.53, size = 128, normalized size = 3.20

$$\frac{\frac{\sin(x)}{\cos(x)+1} + \frac{5\sin(x)^2}{(\cos(x)+1)^2} + \frac{3\sin(x)^3}{(\cos(x)+1)^3} + \frac{3\sin(x)^4}{(\cos(x)+1)^4} + 4}{a + \frac{a\sin(x)}{\cos(x)+1} + \frac{2a\sin(x)^2}{(\cos(x)+1)^2} + \frac{2a\sin(x)^3}{(\cos(x)+1)^3} + \frac{a\sin(x)^4}{(\cos(x)+1)^4} + \frac{a\sin(x)^5}{(\cos(x)+1)^5}} + \frac{3\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*csc(x)),x, algorithm="maxima")
```

```
[Out] (sin(x)/(cos(x) + 1) + 5*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^3/(cos(x) + 1)^
3 + 3*sin(x)^4/(cos(x) + 1)^4 + 4)/(a + a*sin(x)/(cos(x) + 1) + 2*a*sin(x)^
2/(cos(x) + 1)^2 + 2*a*sin(x)^3/(cos(x) + 1)^3 + a*sin(x)^4/(cos(x) + 1)^4
+ a*sin(x)^5/(cos(x) + 1)^5) + 3*arctan(sin(x)/(cos(x) + 1))/a
```

Fricas [A]
time = 5.06, size = 53, normalized size = 1.32

$$\frac{\cos(x)^3 + 3(x+1)\cos(x) + 2\cos(x)^2 - \left(\cos(x)^2 - 3x - \cos(x) + 2\right)\sin(x) + 3x + 2}{2\left(a\cos(x) + a\sin(x) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*csc(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)^3 + 3*(x + 1)*cos(x) + 2*cos(x)^2 - (cos(x)^2 - 3*x - cos(x) +
2)*sin(x) + 3*x + 2)/(a*cos(x) + a*sin(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(x)}{\csc(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*csc(x)),x)
```

```
[Out] Integral(sin(x)**2/(csc(x) + 1), x)/a
```

Giac [A]
time = 0.42, size = 56, normalized size = 1.40

$$\frac{3x}{2a} + \frac{\tan\left(\frac{1}{2}x\right)^3 + 2\tan\left(\frac{1}{2}x\right)^2 - \tan\left(\frac{1}{2}x\right) + 2}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)^2 a} + \frac{2}{a\left(\tan\left(\frac{1}{2}x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*csc(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + (tan(1/2*x)^3 + 2*tan(1/2*x)^2 - tan(1/2*x) + 2)/((tan(1/2*x)^2 +
 1)^2*a) + 2/(a*(tan(1/2*x) + 1))
```

Mupad [B]
time = 0.27, size = 59, normalized size = 1.48

$$\frac{3\,x}{2\,a}+\frac{3\tan\left(\frac{x}{2}\right)^4+3\tan\left(\frac{x}{2}\right)^3+5\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)+4}{a\,\left({\tan\left(\frac{x}{2}\right)}^2+1\right)^2\,\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + a/sin(x)),x)
```

```
[Out] (3*x)/(2*a) + (tan(x/2) + 5*tan(x/2)^2 + 3*tan(x/2)^3 + 3*tan(x/2)^4 + 4)/(
a*(tan(x/2)^2 + 1)^2*(tan(x/2) + 1))
```

3.9 $\int \frac{\sin^3(x)}{a+a\csc(x)}\,dx$

Optimal. Leaf size=53

$$-\frac{3x}{2a}-\frac{4\cos(x)}{a}+\frac{4\cos^3(x)}{3a}+\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^2(x)}{a+a\csc(x)}$$

```
[Out] -3/2*x/a-4*cos(x)/a+4/3*cos(x)^3/a+3/2*cos(x)*sin(x)/a+cos(x)*sin(x)^2/(a+a
*csc(x))
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3904, 3872, 2713, 2715, 8}

$$-\frac{3x}{2a}+\frac{4\cos^3(x)}{3a}-\frac{4\cos(x)}{a}+\frac{3\sin(x)\cos(x)}{2a}+\frac{\sin^2(x)\cos(x)}{a\csc(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Csc[x]),x]
```

```
[Out] (-3*x)/(2*a) - (4*Cos[x])/a + (4*Cos[x]^3)/(3*a) + (3*Cos[x]*Sin[x])/(2*a)
+ (Cos[x]*Sin[x]^2)/(a + a*Csc[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{a+a\csc(x)}\,dx &= \frac{\cos(x)\sin^2(x)}{a+a\csc(x)}-\frac{\int(-4a+3a\csc(x))\sin^3(x)\,dx}{a^2}\\
&= \frac{\cos(x)\sin^2(x)}{a+a\csc(x)}-\frac{3\int\sin^2(x)\,dx}{a}+\frac{4\int\sin^3(x)\,dx}{a}\\
&= \frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^2(x)}{a+a\csc(x)}-\frac{3\int 1\,dx}{2a}-\frac{4\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos(x)\right)}{a}\\
&= -\frac{3x}{2a}-\frac{4\cos(x)}{a}+\frac{4\cos^3(x)}{3a}+\frac{3\cos(x)\sin(x)}{2a}+\frac{\cos(x)\sin^2(x)}{a+a\csc(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 49, normalized size = 0.92

$$
\frac{-21\cos(x)+\cos(3x)+3\left(-6x+\frac{8\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}+\sin(2x)\right)}{12a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Csc[x]),x]
[Out] (-21*Cos[x] + Cos[3*x] + 3*(-6*x + (8*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + Sin
[2*x]))/(12*a)
```

Maple [A]
time = 0.08, size = 66, normalized size = 1.25

method	result
risch	$-\frac{3x}{2a}-\frac{7\,e^{ix}}{8a}-\frac{7\,e^{-ix}}{8a}-\frac{2}{\left(i+e^{ix}\right)a}+\frac{\cos(3x)}{12a}+\frac{\sin(2x)}{4a}$
default	$\frac{-\frac{2}{\tan\left(\frac{x}{2}\right)+1}-\frac{2\left(\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{2}+\tan^4\left(\frac{x}{2}\right)+4\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{5}{3}\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-3\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$
norman	$\frac{-\frac{5\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}+\frac{5\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}-\frac{3x}{2a}-\frac{8}{3a}-\frac{3x\tan\left(\frac{x}{2}\right)}{2a}-\frac{9x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2a}-\frac{9x\left(\tan^3\left(\frac{x}{2}\right)\right)}{2a}-\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2a}-\frac{9x\left(\tan^5\left(\frac{x}{2}\right)\right)}{2a}-\frac{3x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2a}-\frac{3x(t}{}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3\left(\tan\left(\frac{x}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/a*(-1/16/(tan(1/2*x)+1)-1/16*(1/2*tan(1/2*x)^5+tan(1/2*x)^4+4*tan(1/2*x)
^2-1/2*tan(1/2*x)+5/3)/(tan(1/2*x)^2+1)^3-3/32*arctan(tan(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(47) = 94$.
time = 0.47, size = 180, normalized size = 3.40

$$-\frac{\frac{7\sin(x)}{\cos(x)+1}+\frac{39\sin(x)^2}{(\cos(x)+1)^2}+\frac{24\sin(x)^3}{(\cos(x)+1)^3}+\frac{24\sin(x)^4}{(\cos(x)+1)^4}+\frac{9\sin(x)^5}{(\cos(x)+1)^5}+\frac{9\sin(x)^6}{(\cos(x)+1)^6}+16}{3\left(a+\frac{a\sin(x)}{\cos(x)+1}+\frac{3a\sin(x)^2}{(\cos(x)+1)^2}+\frac{3a\sin(x)^3}{(\cos(x)+1)^3}+\frac{3a\sin(x)^4}{(\cos(x)+1)^4}+\frac{3a\sin(x)^5}{(\cos(x)+1)^5}+\frac{a\sin(x)^6}{(\cos(x)+1)^6}+\frac{a\sin(x)^7}{(\cos(x)+1)^7}\right)}-\frac{3\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*csc(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(7*sin(x)/(cos(x) + 1) + 39*sin(x)^2/(cos(x) + 1)^2 + 24*sin(x)^3/(cos
(x) + 1)^3 + 24*sin(x)^4/(cos(x) + 1)^4 + 9*sin(x)^5/(cos(x) + 1)^5 + 9*sin
(x)^6/(cos(x) + 1)^6 + 16)/(a + a*sin(x)/(cos(x) + 1) + 3*a*sin(x)^2/(cos(x
) + 1)^2 + 3*a*sin(x)^3/(cos(x) + 1)^3 + 3*a*sin(x)^4/(cos(x) + 1)^4 + 3*a*
sin(x)^5/(cos(x) + 1)^5 + a*sin(x)^6/(cos(x) + 1)^6 + a*sin(x)^7/(cos(x) +
1)^7) - 3*arctan(sin(x)/(cos(x) + 1))/a
```

Fricas [A]
time = 3.44, size = 70, normalized size = 1.32

$$\frac{2\cos(x)^4-\cos(x)^3-3(3x+5)\cos(x)-12\cos(x)^2+(2\cos(x)^3+3\cos(x)^2-9x-9\cos(x)+6)\sin(x)-9x-6}{6(a\cos(x)+a\sin(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*csc(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(x)^4 - cos(x)^3 - 3*(3*x + 5)*cos(x) - 12*cos(x)^2 + (2*cos(x)^3
 + 3*cos(x)^2 - 9*x - 9*cos(x) + 6)*sin(x) - 9*x - 6)/(a*cos(x) + a*sin(x)
+ a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^3(x)}{\csc(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*csc(x)),x)
```

```
[Out] Integral(sin(x)**3/(csc(x) + 1), x)/a
```

Giac [A]
time = 0.43, size = 67, normalized size = 1.26

$$-\frac{3x}{2a}-\frac{2}{a\left(\tan\left(\frac{1}{2}x\right)+1\right)}-\frac{3\tan\left(\frac{1}{2}x\right)^{5}+6\tan\left(\frac{1}{2}x\right)^{4}+24\tan\left(\frac{1}{2}x\right)^{2}-3\tan\left(\frac{1}{2}x\right)+10}{3\left(\tan\left(\frac{1}{2}x\right)^{2}+1\right)^{3}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*csc(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 2/(a*(tan(1/2*x) + 1)) - 1/3*(3*tan(1/2*x)^5 + 6*tan(1/2*x)^4 +
24*tan(1/2*x)^2 - 3*tan(1/2*x) + 10)/((tan(1/2*x)^2 + 1)^3*a)
```

Mupad [B]
time = 0.35, size = 78, normalized size = 1.47

$$-\frac{3x}{2a}-\frac{3\tan\left(\frac{x}{2}\right)^{6}+3\tan\left(\frac{x}{2}\right)^{5}+8\tan\left(\frac{x}{2}\right)^{4}+8\tan\left(\frac{x}{2}\right)^{3}+13\tan\left(\frac{x}{2}\right)^{2}+\frac{7\tan\left(\frac{x}{2}\right)}{3}+\frac{16}{3}}{a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{3}\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + a/sin(x)),x)
```

```
[Out] - (3*x)/(2*a) - ((7*tan(x/2))/3 + 13*tan(x/2)^2 + 8*tan(x/2)^3 + 8*tan(x/2)
^4 + 3*tan(x/2)^5 + 3*tan(x/2)^6 + 16/3)/(a*(tan(x/2)^2 + 1)^3*(tan(x/2) +
1))
```

3.10 $\int \frac{\sin^4(x)}{a+a\csc(x)}\,dx$

Optimal. Leaf size=66

$$\frac{15x}{8a}+\frac{4\cos(x)}{a}-\frac{4\cos^3(x)}{3a}-\frac{15\cos(x)\sin(x)}{8a}-\frac{5\cos(x)\sin^3(x)}{4a}+\frac{\cos(x)\sin^3(x)}{a+a\csc(x)}$$

```
[Out] 15/8*x/a+4*cos(x)/a-4/3*cos(x)^3/a-15/8*cos(x)*sin(x)/a-5/4*cos(x)*sin(x)^3
/a+cos(x)*sin(x)^3/(a+a*csc(x))
```

Rubi [A]
time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3904, 3872, 2715, 8, 2713}

$$\frac{15x}{8a}-\frac{4\cos^3(x)}{3a}+\frac{4\cos(x)}{a}-\frac{5\sin^3(x)\cos(x)}{4a}-\frac{15\sin(x)\cos(x)}{8a}+\frac{\sin^3(x)\cos(x)}{a\csc(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Csc[x]),x]
```

```
[Out] (15*x)/(8*a) + (4*Cos[x])/a - (4*Cos[x]^3)/(3*a) - (15*Cos[x]*Sin[x])/(8*a)
 - (5*Cos[x]*Sin[x]^3)/(4*a) + (Cos[x]*Sin[x]^3)/(a + a*Csc[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(x)}{a+a\csc(x)}\,dx &= \frac{\cos(x)\sin^3(x)}{a+a\csc(x)} - \frac{\int(-5a+4a\csc(x))\sin^4(x)\,dx}{a^2} \\
&= \frac{\cos(x)\sin^3(x)}{a+a\csc(x)} - \frac{4\int\sin^3(x)\,dx}{a} + \frac{5\int\sin^4(x)\,dx}{a} \\
&= -\frac{5\cos(x)\sin^3(x)}{4a} + \frac{\cos(x)\sin^3(x)}{a+a\csc(x)} + \frac{15\int\sin^2(x)\,dx}{4a} + \frac{4\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos(
\end{aligned}$$

$$\begin{aligned}
&= \frac{4\cos(x)}{a} - \frac{4\cos^3(x)}{3a} - \frac{15\cos(x)\sin(x)}{8a} - \frac{5\cos(x)\sin^3(x)}{4a} + \frac{\cos(x)\sin^3(x)}{a+a\csc(x)} + \frac{15\int 1\,d}{8a} \\
&= \frac{15x}{8a} + \frac{4\cos(x)}{a} - \frac{4\cos^3(x)}{3a} - \frac{15\cos(x)\sin(x)}{8a} - \frac{5\cos(x)\sin^3(x)}{4a} + \frac{\cos(x)\sin^3(x)}{a+a\csc(x)}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 57, normalized size = 0.86

$$\frac{168\cos(x)-8\cos(3x)+3\left(60x-\frac{64\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}-16\sin(2x)+\sin(4x)\right)}{96a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Csc[x]),x]
[Out] (168*Cos[x] - 8*Cos[3*x] + 3*(60*x - (64*Sin[x/2])/(Cos[x/2] + Sin[x/2]) -
16*Sin[2*x] + Sin[4*x]))/(96*a)
```

Maple [A]
time = 0.08, size = 90, normalized size = 1.36

method	result
risch	$\frac{15x}{8a}+\frac{7\,\mathrm{e}^{ix}}{8a}+\frac{7\,\mathrm{e}^{-ix}}{8a}+\frac{2}{(i+\mathrm{e}^{ix})a}+\frac{\sin(4x)}{32a}-\frac{\cos(3x)}{12a}-\frac{\sin(2x)}{2a}$
default	$\frac{\frac{64}{32\tan\left(\frac{x}{2}\right)+32}+\frac{2\left(\frac{7\left(\tan^7\left(\frac{x}{2}\right)\right)}{8}+\tan^6\left(\frac{x}{2}\right)+\frac{15\left(\tan^5\left(\frac{x}{2}\right)\right)}{8}+5\left(\tan^4\left(\frac{x}{2}\right)\right)-\frac{15\left(\tan^3\left(\frac{x}{2}\right)\right)}{8}+\frac{17\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}-\frac{7\tan\left(\frac{x}{2}\right)}{8}+\frac{5}{3}\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^4}+\frac{15\arctan\left(\tan\left(\frac{x}{2}\right.}{4}}{a}$

norman	$\frac{\frac{15x}{8a}+\frac{15}{4a}+\frac{15x\tan\left(\frac{x}{2}\right)}{8a}+\frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2a}+\frac{15x\left(\tan^3\left(\frac{x}{2}\right)\right)}{2a}+\frac{45x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4a}+\frac{45x\left(\tan^5\left(\frac{x}{2}\right)\right)}{4a}+\frac{15x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2a}+\frac{15x\left(\tan^7\left(\frac{x}{2}\right)\right)}{2a}+\frac{15x\left(\tan^8\left(\frac{x}{2}\right)\right.}{8a}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+a*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 64/a*(1/32/(tan(1/2*x)+1)+1/32*(7/8*tan(1/2*x)^7+tan(1/2*x)^6+15/8*tan(1/2*
x)^5+5*tan(1/2*x)^4-15/8*tan(1/2*x)^3+17/3*tan(1/2*x)^2-7/8*tan(1/2*x)+5/3)
/(tan(1/2*x)^2+1)^4+15/256*arctan(tan(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(58) = 116$.
time = 0.48, size = 230, normalized size = 3.48

$$\frac{\frac{19\sin(x)}{\cos(x)+1}+\frac{211\sin(x)^2}{(\cos(x)+1)^2}+\frac{91\sin(x)^3}{(\cos(x)+1)^3}+\frac{219\sin(x)^4}{(\cos(x)+1)^4}+\frac{165\sin(x)^5}{(\cos(x)+1)^5}+\frac{165\sin(x)^6}{(\cos(x)+1)^6}+\frac{45\sin(x)^7}{(\cos(x)+1)^7}+\frac{45\sin(x)^8}{(\cos(x)+1)^8}+64}{12\left(a+\frac{a\sin(x)}{\cos(x)+1}+\frac{4a\sin(x)^2}{(\cos(x)+1)^2}+\frac{4a\sin(x)^3}{(\cos(x)+1)^3}+\frac{6a\sin(x)^4}{(\cos(x)+1)^4}+\frac{6a\sin(x)^5}{(\cos(x)+1)^5}+\frac{4a\sin(x)^6}{(\cos(x)+1)^6}+\frac{4a\sin(x)^7}{(\cos(x)+1)^7}+\frac{a\sin(x)^8}{(\cos(x)+1)^8}+\frac{a\sin(x)^9}{(\cos(x)+1)^9}\right)}+\frac{15\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*csc(x)),x, algorithm="maxima")
```

```
[Out] 1/12*(19*sin(x)/(cos(x) + 1) + 211*sin(x)^2/(cos(x) + 1)^2 + 91*sin(x)^3/(c
os(x) + 1)^3 + 219*sin(x)^4/(cos(x) + 1)^4 + 165*sin(x)^5/(cos(x) + 1)^5 +
165*sin(x)^6/(cos(x) + 1)^6 + 45*sin(x)^7/(cos(x) + 1)^7 + 45*sin(x)^8/(cos
(x) + 1)^8 + 64)/(a + a*sin(x)/(cos(x) + 1) + 4*a*sin(x)^2/(cos(x) + 1)^2 +
 4*a*sin(x)^3/(cos(x) + 1)^3 + 6*a*sin(x)^4/(cos(x) + 1)^4 + 6*a*sin(x)^5/(
cos(x) + 1)^5 + 4*a*sin(x)^6/(cos(x) + 1)^6 + 4*a*sin(x)^7/(cos(x) + 1)^7 +
 a*sin(x)^8/(cos(x) + 1)^8 + a*sin(x)^9/(cos(x) + 1)^9) + 15/4*arctan(sin(x
)/(cos(x) + 1))/a
```

Fricas [A]
time = 3.89, size = 81, normalized size = 1.23

$$-\frac{6\cos(x)^5+8\cos(x)^4-25\cos(x)^3-45(x+1)\cos(x)-48\cos(x)^2-\left(6\cos(x)^4-2\cos(x)^3-27\cos(x)^2+45x+21\cos(x)-24\right)\sin(x)-45x-24}{24\left(a\cos(x)+a\sin(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*csc(x)),x, algorithm="fricas")
```

```
[Out] -1/24*(6*cos(x)^5 + 8*cos(x)^4 - 25*cos(x)^3 - 45*(x + 1)*cos(x) - 48*cos(x
)^2 - (6*cos(x)^4 - 2*cos(x)^3 - 27*cos(x)^2 + 45*x + 21*cos(x) - 24)*sin(x
) - 45*x - 24)/(a*cos(x) + a*sin(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^4(x)}{\csc(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*csc(x)),x)
```

```
[Out] Integral(sin(x)**4/(csc(x) + 1), x)/a
```

Giac [A]
time = 0.40, size = 91, normalized size = 1.38

$$\frac{15\,x}{8\,a}+\frac{2}{a\left(\tan\left(\frac{1}{2}\,x\right)+1\right)}+\frac{21\,\tan\left(\frac{1}{2}\,x\right)^{7}+24\,\tan\left(\frac{1}{2}\,x\right)^{6}+45\,\tan\left(\frac{1}{2}\,x\right)^{5}+120\,\tan\left(\frac{1}{2}\,x\right)^{4}-45\,\tan\left(\frac{1}{2}\,x\right)^{3}+136\,\tan\left(\frac{1}{2}\,x\right)^{2}-21\,\tan\left(\frac{1}{2}\,x\right)+40}{12\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)^{4}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*csc(x)),x, algorithm="giac")
```

```
[Out] 15/8*x/a + 2/(a*(tan(1/2*x) + 1)) + 1/12*(21*tan(1/2*x)^7 + 24*tan(1/2*x)^6
 + 45*tan(1/2*x)^5 + 120*tan(1/2*x)^4 - 45*tan(1/2*x)^3 + 136*tan(1/2*x)^2
- 21*tan(1/2*x) + 40)/((tan(1/2*x)^2 + 1)^4*a)
```

Mupad [B]
time = 0.40, size = 93, normalized size = 1.41

$$\frac{15\,x}{8\,a}+\frac{\frac{15\tan\left(\frac{x}{2}\right)^{8}}{4}+\frac{15\tan\left(\frac{x}{2}\right)^{7}}{4}+\frac{55\tan\left(\frac{x}{2}\right)^{6}}{4}+\frac{55\tan\left(\frac{x}{2}\right)^{5}}{4}+\frac{73\tan\left(\frac{x}{2}\right)^{4}}{4}+\frac{91\tan\left(\frac{x}{2}\right)^{3}}{12}+\frac{211\tan\left(\frac{x}{2}\right)^{2}}{12}+\frac{19\tan\left(\frac{x}{2}\right)}{12}+\frac{16}{3}}{a\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{4}\,\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + a/sin(x)),x)
```

```
[Out] (15*x)/(8*a) + ((19*tan(x/2))/12 + (211*tan(x/2)^2)/12 + (91*tan(x/2)^3)/12
 + (73*tan(x/2)^4)/4 + (55*tan(x/2)^5)/4 + (55*tan(x/2)^6)/4 + (15*tan(x/2)
^7)/4 + (15*tan(x/2)^8)/4 + 16/3)/(a*(tan(x/2)^2 + 1)^4*(tan(x/2) + 1))
```

3.11 $\int \frac{1}{(a+a\csc(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$\frac{x}{a^2}+\frac{4\cot(c+dx)}{3a^2d(1+\csc(c+dx))}+\frac{\cot(c+dx)}{3d(a+a\csc(c+dx))^2}$$

```
[Out] x/a^2+4/3*cot(d*x+c)/a^2/d/(1+csc(d*x+c))+1/3*cot(d*x+c)/d/(a+a*csc(d*x+c))
^2
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{3862, 4004, 3879\}$

$$\frac{4\cot(c+dx)}{3a^2d(\csc(c+dx)+1)}+\frac{x}{a^2}+\frac{\cot(c+dx)}{3d(a\csc(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Csc[c + d*x])^(-2),x]
```

```
[Out] x/a^2 + (4*Cot[c + d*x])/(3*a^2*d*(1 + Csc[c + d*x])) + Cot[c + d*x]/(3*d*(
a + a*Csc[c + d*x])^2)
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\csc(c+dx))^2}\,dx = \frac{\cot(c+dx)}{3d(a+a\csc(c+dx))^2} - \frac{\int \frac{-3a+a\csc(c+dx)}{a+a\csc(c+dx)}\,dx}{3a^2}$$
$$= \frac{x}{a^2} + \frac{\cot(c+dx)}{3d(a+a\csc(c+dx))^2} - \frac{4\int \frac{\csc(c+dx)}{a+a\csc(c+dx)}\,dx}{3a}$$
$$= \frac{x}{a^2} + \frac{\cot(c+dx)}{3d(a+a\csc(c+dx))^2} + \frac{4\cot(c+dx)}{3d\,(a^2+a^2\csc(c+dx))}$$

Mathematica [A]
time = 0.34, size = 108, normalized size = 1.89

$$\frac{3(-4+3c+3dx)\cos\left(\frac{1}{2}(c+dx)\right)+(10-3c-3dx)\cos\left(\frac{3}{2}(c+dx)\right)+6(-3+2c+2dx+(c+dx)\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)}{6a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Csc[c + d*x])^(-2),x]
```

```
[Out] (3*(-4 + 3*c + 3*d*x)*Cos[(c + d*x)/2] + (10 - 3*c - 3*d*x)*Cos[(3*(c + d*x
))/2] + 6*(-3 + 2*c + 2*d*x + (c + d*x)*Cos[c + d*x])*Sin[(c + d*x)/2])/(6*
a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A]
time = 0.11, size = 67, normalized size = 1.18

method	result	size
risch	$\frac{x}{a^2}+\frac{6i e^{i(dx+c)}+4\,e^{2i(dx+c)}-\frac{10}{3}}{d\,a^2\left(i+e^{i(dx+c)}\right)^3}$	54
derivativedivides	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4}}{a^2d}$	67
default	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4}}{a^2d}$	67
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2}{3ad}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*csc(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^2*(1/4*arctan(tan(1/2*d*x+1/2*c))-1/6/(tan(1/2*d*x+1/2*c)+1)^3+1/4/(t
an(1/2*d*x+1/2*c)+1)^2+1/4/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(53) = 106$.
time = 0.49, size = 142, normalized size = 2.49

$$\frac{2\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+4}{a^2+\frac{3a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 4)/(a^2 + 3*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*arct
an(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.
time = 3.01, size = 124, normalized size = 2.18

$$\frac{(3\,dx-5)\cos{(dx+c)}^2-6\,dx-(3\,dx+4)\cos{(dx+c)}-(6\,dx+(3\,dx+5)\cos{(dx+c)}+1)\sin{(dx+c)}+1}{3\left(a^2d\cos{(dx+c)}^2-a^2d\cos{(dx+c)}-2\,a^2d-(a^2d\cos{(dx+c)}+2\,a^2d)\sin{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*d*x - 5)*cos(d*x + c)^2 - 6*d*x - (3*d*x + 4)*cos(d*x + c) - (6*d*x
 + (3*d*x + 5)*cos(d*x + c) + 1)*sin(d*x + c) + 1)/(a^2*d*cos(d*x + c)^2 -
a^2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\csc^2{(c+dx)}+2\csc{(c+dx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c))**2,x)
```

```
[Out] Integral(1/(csc(c + d*x)**2 + 2*csc(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.44, size = 60, normalized size = 1.05

$$\frac{\frac{3\,(dx+c)}{a^2}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)/a^2 + 2*(3*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c)
 + 4)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 0.41, size = 52, normalized size = 0.91

$$\frac{x}{a^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{8}{3}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/sin(c + d*x))^2,x)
```

```
[Out] x/a^2 + (6*tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^2 + 8/3)/(a^2*d*(tan(c
/2 + (d*x)/2) + 1)^3)
```

3.12 $\int \frac{1}{(a+a\csc(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{x}{a^3}+\frac{\cot(c+dx)}{5d(a+a\csc(c+dx))^3}+\frac{7\cot(c+dx)}{15ad(a+a\csc(c+dx))^2}+\frac{22\cot(c+dx)}{15d\left(a^3+a^3\csc(c+dx)\right)}$$

```
[Out] x/a^3+1/5*cot(d*x+c)/d/(a+a*csc(d*x+c))^3+7/15*cot(d*x+c)/a/d/(a+a*csc(d*x+
c))^2+22/15*cot(d*x+c)/d/(a^3+a^3*csc(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3862, 4007, 4004, 3879}

$$\frac{22\cot(c+dx)}{15d\left(a^3\csc(c+dx)+a^3\right)}+\frac{x}{a^3}+\frac{7\cot(c+dx)}{15ad(a\csc(c+dx)+a)^2}+\frac{\cot(c+dx)}{5d(a\csc(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Csc[c + d*x])^(-3),x]
```

```
[Out] x/a^3 + Cot[c + d*x]/(5*d*(a + a*Csc[c + d*x])^3) + (7*Cot[c + d*x])/(15*a*
d*(a + a*Csc[c + d*x])^2) + (22*Cot[c + d*x])/(15*d*(a^3 + a^3*Csc[c + d*x]
))
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+a\csc(c+dx))^3}\,dx = \frac{\cot(c+dx)}{5d(a+a\csc(c+dx))^3} - \frac{\int \frac{-5a+2a\csc(c+dx)}{(a+a\csc(c+dx))^2}\,dx}{5a^2}$$
$$= \frac{\cot(c+dx)}{5d(a+a\csc(c+dx))^3} + \frac{7\cot(c+dx)}{15ad(a+a\csc(c+dx))^2} + \frac{\int \frac{15a^2-7a^2\csc(c+dx)}{a+a\csc(c+dx)}\,dx}{15a^4}$$
$$= \frac{x}{a^3} + \frac{\cot(c+dx)}{5d(a+a\csc(c+dx))^3} + \frac{7\cot(c+dx)}{15ad(a+a\csc(c+dx))^2} - \frac{22\int \frac{\csc(c+dx)}{a+a\csc(c+dx)}\,dx}{15a^2}$$
$$= \frac{x}{a^3} + \frac{\cot(c+dx)}{5d(a+a\csc(c+dx))^3} + \frac{7\cot(c+dx)}{15ad(a+a\csc(c+dx))^2} + \frac{22\cot(c+dx)}{15d\left(a^3+a^3\csc(c+dx)\right)}$$

Mathematica [A]
time = 0.97, size = 123, normalized size = 1.40

$$\frac{15c+15dx+\frac{3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}-\frac{13}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(-38+16\cos(2(c+dx))-51\sin(c+dx))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}}{15a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Csc[c + d*x])^(-3),x]
[Out] (15*c + 15*d*x + 3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 13/(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-38 + 16*Cos[2*(c + d*x
)] - 51*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(15*a^3*d)
```

Maple [A]
time = 0.13, size = 97, normalized size = 1.10

method	result
risch	$\frac{x}{a^3}+\frac{-\frac{74\,e^{2i(dx+c)}}{3}+18ie^{3i(dx+c)}-\frac{46ie^{i(dx+c)}}{3}+6\,e^{4i(dx+c)}+\frac{64}{15}}{d\,a^3\left(i+e^{i(dx+c)}\right)^5}$

derivativedivides	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{16}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+}}{a^3d}$
default	$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{16}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+}}{a^3d}$
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{10x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{10x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{5x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{44}{15a}}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*csc(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^3*(1/8*arctan(tan(1/2*d*x+1/2*c))-1/4/(tan(1/2*d*x+1/2*c)+1)^4+1/10/
(tan(1/2*d*x+1/2*c)+1)^5+1/12/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan(1/2*d*x+1/2
*c)+1)^2+1/8/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(82) = 164$.
time = 0.47, size = 228, normalized size = 2.59

$$\frac{2\left(\frac{\frac{95\sin(dx+c)}{\cos(dx+c)+1}+\frac{145\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+22}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/15*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(82) = 164$.
time = 3.57, size = 181, normalized size = 2.06

$$\frac{(15\,dx+32)\cos(dx+c)^3+(45\,dx-19)\cos(dx+c)^2-60\,dx-6\,(5\,dx+9)\cos(dx+c)+((15\,dx-32)\cos(dx+c)^2-60\,dx-3\,(10\,dx+17)\cos(dx+c)+3)\sin(dx+c)-3}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d+\left(a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*((15*d*x + 32)*cos(d*x + c)^3 + (45*d*x - 19)*cos(d*x + c)^2 - 60*d*x
- 6*(5*d*x + 9)*cos(d*x + c) + ((15*d*x - 32)*cos(d*x + c)^2 - 60*d*x - 3*(
```

```
10*d*x + 17)*cos(d*x + c) + 3)*sin(d*x + c) - 3)/(a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)
^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\csc^3(c+dx)+3\csc^2(c+dx)+3\csc(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c))**3,x)
```

```
[Out] Integral(1/(csc(c + d*x)**3 + 3*csc(c + d*x)**2 + 3*csc(c + d*x) + 1), x)/a
**3
```

Giac [A]
time = 0.43, size = 86, normalized size = 0.98

$$\frac{\frac{15\,(dx+c)}{a^3}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+145\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+95\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+22\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^4 + 75*tan(1/2*d*x + 1/
2*c)^3 + 145*tan(1/2*d*x + 1/2*c)^2 + 95*tan(1/2*d*x + 1/2*c) + 22)/(a^3*(t
an(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 2.18, size = 78, normalized size = 0.89

$$\frac{x}{a^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{58\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{38\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{44}{15}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/sin(c + d*x))^3,x)
```

```
[Out] x/a^3 + ((38*tan(c/2 + (d*x)/2))/3 + (58*tan(c/2 + (d*x)/2)^2)/3 + 10*tan(c
/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^4 + 44/15)/(a^3*d*(tan(c/2 + (d*x)/2
) + 1)^5)
```

3.13 $\int (a+a\csc(x))^{5/2}\,dx$

Optimal. Leaf size=65

$$-2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\cot(x)}{\sqrt{a+a\csc(x)}}\right)-\frac{14a^3\cot(x)}{3\sqrt{a+a\csc(x)}}-\frac{2}{3}a^2\cot(x)\sqrt{a+a\csc(x)}$$

[Out] -2*a^(5/2)*arctan(cot(x)*a^(1/2)/(a+a*csc(x))^(1/2))-14/3*a^3*cot(x)/(a+a*csc(x))^(1/2)-2/3*a^2*cot(x)*(a+a*csc(x))^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3860, 4000, 3859, 209, 3877}

$$-2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\cot(x)}{\sqrt{a\csc(x)+a}}\right)-\frac{14a^3\cot(x)}{3\sqrt{a\csc(x)+a}}-\frac{2}{3}a^2\cot(x)\sqrt{a\csc(x)+a}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Csc[x])^(5/2),x]

[Out] -2*a^(5/2)*ArcTan[(Sqrt[a]*Cot[x])/Sqrt[a + a*Csc[x]]] - (14*a^3*Cot[x])/(3*Sqrt[a + a*Csc[x]]) - (2*a^2*Cot[x]*Sqrt[a + a*Csc[x]])/3

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d), Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*Cot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1), Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\csc(x))^{5/2}\,dx &= -\frac{2}{3}a^2\cot(x)\sqrt{a+a\csc(x)}+\frac{1}{3}(2a)\int\sqrt{a+a\csc(x)}\left(\frac{3a}{2}+\frac{7}{2}a\csc(x)\right)dx\\ &= -\frac{2}{3}a^2\cot(x)\sqrt{a+a\csc(x)}+a^2\int\sqrt{a+a\csc(x)}\,dx+\frac{1}{3}\left(7a^2\right)\int\csc(x)\sqrt{a+a\\ &= -\frac{14a^3\cot(x)}{3\sqrt{a+a\csc(x)}}-\frac{2}{3}a^2\cot(x)\sqrt{a+a\csc(x)}-\left(2a^3\right)\text{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\right.\\ &= -2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{a+a\csc(x)}}\right)-\frac{14a^3\cot(x)}{3\sqrt{a+a\csc(x)}}-\frac{2}{3}a^2\cot(x)\sqrt{a+a\csc(x)}\end{aligned}$$

Mathematica [**A**]
time = 1.94, size = 80, normalized size = 1.23

$$-\frac{2a^2\sqrt{a(1+\csc(x))}\left(3\text{ArcTan}\left(\sqrt{-1+\csc(x)}\right)+\sqrt{-1+\csc(x)}\,(8+\csc(x))\right)\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{3\sqrt{-1+\csc(x)}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Csc[x])^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[a*(1 + Csc[x])]*(3*ArcTan[Sqrt[-1 + Csc[x]]] + Sqrt[-1 + Csc[x
]]*(8 + Csc[x]))*(Cos[x/2] - Sin[x/2]))/(3*Sqrt[-1 + Csc[x]]*(Cos[x/2] + Si
n[x/2]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(51) = 102$.
time = 0.16, size = 535, normalized size = 8.23

method	result

default	$\frac{\left(-3\sin(x)\cos(x)\ln\left(-\frac{\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\sqrt{2}\,\sin(x)+\sin(x)-\cos(x)+1}{\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\sqrt{2}\,\sin(x)-\sin(x)+\cos(x)-1}\right)\left(-\frac{\cos(x)-1}{\sin(x)}\right)^{\frac{3}{2}}-12\sin(x)\cos(x)\arctan\left(\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*csc(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-3*sin(x)*cos(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)
-cos(x)+1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))*(-(
cos(x)-1)/sin(x))^(3/2)-12*sin(x)*cos(x)*arctan((-(cos(x)-1)/sin(x))^(1/2)*
2^(1/2)+1)*(-(cos(x)-1)/sin(x))^(3/2)-12*sin(x)*cos(x)*arctan((-(cos(x)-1)/
sin(x))^(1/2)*2^(1/2)-1)*(-(cos(x)-1)/sin(x))^(3/2)-3*sin(x)*cos(x)*ln(-((-
(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(
x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1))*(-(cos(x)-1)/sin(x))^(3/2)-3*sin
(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1)/((-(cos
(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))*(-(cos(x)-1)/sin(x))^
(3/2)-12*sin(x)*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)*(-(cos(x)-1)/s
in(x))^(3/2)-12*sin(x)*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)-1)*(-(cos(
x)-1)/sin(x))^(3/2)-3*sin(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)
-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+
1))*(-(cos(x)-1)/sin(x))^(3/2)+16*sin(x)*cos(x)*2^(1/2)-16*cos(x)^2*2^(1/2)
-14*sin(x)*2^(1/2)+2*cos(x)*2^(1/2)+14*2^(1/2))*sin(x)*(a*(sin(x)+1)/sin(x)
)^(5/2)/(sin(x)*cos(x)^2-cos(x)^3+2*cos(x)*sin(x)+3*cos(x)^2-4*sin(x)+2*cos
(x)-4)*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(51) = 102$.
time = 0.48, size = 417, normalized size = 6.42

$$\frac{1}{22}\sqrt{2}a^{\frac{5}{2}}\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{11}{2}}+\frac{5}{18}\sqrt{2}a^{\frac{5}{2}}\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{9}{2}}+\frac{9}{14}\sqrt{2}a^{\frac{5}{2}}\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{7}{2}}+\frac{1}{2}\sqrt{2}a^{\frac{5}{2}}\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{5}{2}}-\frac{2}{3}\sqrt{2}a^{\frac{5}{2}}\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{3}{2}}+\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)\right)\right)a^{\frac{5}{2}}-2\sqrt{2}a^{\frac{5}{2}}\sqrt{\frac{\sin(x)}{\cos(x)+1}}-\frac{\frac{693\sqrt{2}a^{\frac{5}{2}}\sin(x)}{\cos(x)+1}+\frac{1155\sqrt{2}a^{\frac{5}{2}}\sin(x)^2}{(\cos(x)+1)^2}+\frac{1386\sqrt{2}a^{\frac{5}{2}}\sin(x)^3}{(\cos(x)+1)^3}+\frac{990\sqrt{2}a^{\frac{5}{2}}\sin(x)^4}{(\cos(x)+1)^4}+\frac{385\sqrt{2}a^{\frac{5}{2}}\sin(x)^5}{(\cos(x)+1)^5}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(x)^6}{(\cos(x)+1)^6}}{1386\sqrt{\frac{\sin(x)}{\cos(x)+1}}}-\frac{\frac{7\sqrt{2}a^{\frac{5}{2}}\sin(x)}{\cos(x)+1}+\frac{105\sqrt{2}a^{\frac{5}{2}}\sin(x)^2}{(\cos(x)+1)^2}-\frac{210\sqrt{2}a^{\frac{5}{2}}\sin(x)^3}{(\cos(x)+1)^3}-\frac{70\sqrt{2}a^{\frac{5}{2}}\sin(x)^4}{(\cos(x)+1)^4}-\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(x)^5}{(\cos(x)+1)^5}-\frac{3\sqrt{2}a^{\frac{5}{2}}\sin(x)^6}{(\cos(x)+1)^6}}{42\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/22*sqrt(2)*a^(5/2)*(sin(x)/(cos(x) + 1))^(11/2) + 5/18*sqrt(2)*a^(5/2)*(s
in(x)/(cos(x) + 1))^(9/2) + 9/14*sqrt(2)*a^(5/2)*(sin(x)/(cos(x) + 1))^(7/2
) + 1/2*sqrt(2)*a^(5/2)*(sin(x)/(cos(x) + 1))^(5/2) - 2/3*sqrt(2)*a^(5/2)*(
sin(x)/(cos(x) + 1))^(3/2) + sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2*sqrt(sin(x)/(cos(x) + 1)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*s
qrt(sin(x)/(cos(x) + 1)))))*a^(5/2) - 2*sqrt(2)*a^(5/2)*sqrt(sin(x)/(cos(x)
 + 1)) - 1/1386*(693*sqrt(2)*a^(5/2)*sin(x)/(cos(x) + 1) + 1155*sqrt(2)*a^(
5/2)*sin(x)^2/(cos(x) + 1)^2 + 1386*sqrt(2)*a^(5/2)*sin(x)^3/(cos(x) + 1)^3
 + 990*sqrt(2)*a^(5/2)*sin(x)^4/(cos(x) + 1)^4 + 385*sqrt(2)*a^(5/2)*sin(x)
```

```
^5/(cos(x) + 1)^5 + 63*sqrt(2)*a^(5/2)*sin(x)^6/(cos(x) + 1)^6)/sqrt(sin(x)
/(cos(x) + 1)) - 1/42*(7*sqrt(2)*a^(5/2)*sin(x)/(cos(x) + 1) + 105*sqrt(2)*
a^(5/2)*sin(x)^2/(cos(x) + 1)^2 - 210*sqrt(2)*a^(5/2)*sin(x)^3/(cos(x) + 1)
^3 - 70*sqrt(2)*a^(5/2)*sin(x)^4/(cos(x) + 1)^4 - 21*sqrt(2)*a^(5/2)*sin(x)
^5/(cos(x) + 1)^5 - 3*sqrt(2)*a^(5/2)*sin(x)^6/(cos(x) + 1)^6)/(sin(x)/(cos
(x) + 1))^(5/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(51) = 102$.
time = 5.05, size = 318, normalized size = 4.89

$$\left[\frac{3\left(a^{2}\cos\left(x\right)^{2}-a^{2}-\left(a^{2}\cos\left(x\right)+a^{2}\right)\sin\left(x\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(x)^{2}-2\left(\cos(x)^{2}+(\cos(x)+1)\sin(x)-1\right)\sqrt{-a}\sqrt{\frac{a\sin\left(x\right)+a}{\sin\left(x\right)}}+a\cos(x)-(2\,a\cos(x)+a)\sin(x)-a}{\cos(x)+\sin(x)+1}\right)+2\left(8\,a^{2}\cos\left(x\right)^{2}+a^{2}\cos\left(x\right)-7\,a^{2}+\left(8\,a^{2}\cos\left(x\right)+7\,a^{2}\right)\sin\left(x\right)\right)\sqrt{\frac{a\sin\left(x\right)+a}{\sin\left(x\right)}}}{3\left(\cos\left(x\right)^{2}-\left(\cos\left(x\right)+1\right)\sin\left(x\right)-1\right)}, \frac{2\left(3\left(a^{2}\cos\left(x\right)^{2}-a^{2}-\left(a^{2}\cos\left(x\right)+a^{2}\right)\sin\left(x\right)\right)\sqrt{a}\arctan\left(-\frac{\sqrt{a}\sqrt{\frac{a\sin\left(x\right)+a}{\sin\left(x\right)}}(\cos(x)-\sin(x)+1)}{a\cos(x)+a\sin(x)+a}\right)+\left(8\,a^{2}\cos\left(x\right)^{2}+a^{2}\cos\left(x\right)-7\,a^{2}+\left(8\,a^{2}\cos\left(x\right)+7\,a^{2}\right)\sin\left(x\right)\right)\sqrt{\frac{a\sin\left(x\right)+a}{\sin\left(x\right)}}\right)}{3\left(\cos\left(x\right)^{2}-\left(\cos\left(x\right)+1\right)\sin\left(x\right)-1\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*cos(x)^2 - a^2 - (a^2*cos(x) + a^2)*sin(x))*sqrt(-a)*log((2*a*
cos(x)^2 - 2*(cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt(-a)*sqrt((a*sin(x) +
 a)/sin(x)) + a*cos(x) - (2*a*cos(x) + a)*sin(x) - a)/(cos(x) + sin(x) + 1)
) + 2*(8*a^2*cos(x)^2 + a^2*cos(x) - 7*a^2 + (8*a^2*cos(x) + 7*a^2)*sin(x))
*sqrt((a*sin(x) + a)/sin(x)))/(cos(x)^2 - (cos(x) + 1)*sin(x) - 1), 2/3*(3*
(a^2*cos(x)^2 - a^2 - (a^2*cos(x) + a^2)*sin(x))*sqrt(a)*arctan(-sqrt(a)*sq
rt((a*sin(x) + a)/sin(x))*(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a))
+ (8*a^2*cos(x)^2 + a^2*cos(x) - 7*a^2 + (8*a^2*cos(x) + 7*a^2)*sin(x))*sqr
t((a*sin(x) + a)/sin(x)))/(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\csc\left(x\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))**(5/2),x)
```

```
[Out] Integral((a*csc(x) + a)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(51) = 102$.
time = 0.58, size = 250, normalized size = 3.85

$$\left(a^{2}\sqrt{|a|}+a|a|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}+2\sqrt{a\tan\left(\frac{1}{2}\,x\right)}\right)}{2\sqrt{|a|}}\right)+\left(a^{2}\sqrt{|a|}+a|a|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}-2\sqrt{a\tan\left(\frac{1}{2}\,x\right)}\right)}{2\sqrt{|a|}}\right)+\frac{1}{2}\left(a^{2}\sqrt{|a|}-a|a|^{\frac{3}{2}}\right)\log\left(a\tan\left(\frac{1}{2}\,x\right)+\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,x\right)}\sqrt{|a|}+|a|\right)-\frac{1}{2}\left(a^{2}\sqrt{|a|}-a|a|^{\frac{3}{2}}\right)\log\left(a\tan\left(\frac{1}{2}\,x\right)-\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,x\right)}\sqrt{|a|}+|a|\right)+\frac{1}{6}\sqrt{2}\left(\sqrt{a\tan\left(\frac{1}{2}\,x\right)}a^{2}\tan\left(\frac{1}{2}\,x\right)+15\sqrt{a\tan\left(\frac{1}{2}\,x\right)}a^{2}\right)-\frac{\sqrt{2}\left(15\,a^{4}\tan\left(\frac{1}{2}\,x\right)+a^{4}\right)}{6\sqrt{a\tan\left(\frac{1}{2}\,x\right)}a\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(5/2),x, algorithm="giac")
```

```
[Out] (a^2*sqrt(abs(a)) + a*abs(a)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)
) + 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a))) + (a^2*sqrt(abs(a)) + a*abs(a)^(3/2
))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) - 2*sqrt(a*tan(1/2*x)))/sqrt(a
bs(a))) + 1/2*(a^2*sqrt(abs(a)) - a*abs(a)^(3/2))*log(a*tan(1/2*x) + sqrt(2
)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a)) - 1/2*(a^2*sqrt(abs(a)) - a*abs
(a)^(3/2))*log(a*tan(1/2*x) - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs
(a)) + 1/6*sqrt(2)*(sqrt(a*tan(1/2*x))*a^2*tan(1/2*x) + 15*sqrt(a*tan(1/2*x
))*a^2) - 1/6*sqrt(2)*(15*a^4*tan(1/2*x) + a^4)/(sqrt(a*tan(1/2*x))*a*tan(1
/2*x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a+\frac{a}{\sin(x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(x))^(5/2),x)
```

```
[Out] int((a + a/sin(x))^(5/2), x)
```

3.14 $\int (a + a\csc(x))^{3/2}\,dx$

Optimal. Leaf size=44

$$-2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\cot(x)}{\sqrt{a+a\csc(x)}}\right)-\frac{2a^2\cot(x)}{\sqrt{a+a\csc(x)}}$$

```
[Out] -2*a^(3/2)*arctan(cot(x)*a^(1/2)/(a+a*csc(x))^(1/2))-2*a^2*cot(x)/(a+a*csc(
x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3860, 21, 3859, 209}

$$-2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\cot(x)}{\sqrt{a\csc(x)+a}}\right)-\frac{2a^2\cot(x)}{\sqrt{a\csc(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Csc[x])^(3/2),x]
```

```
[Out] -2*a^(3/2)*ArcTan[(Sqrt[a]*Cot[x])/Sqrt[a + a*Csc[x]]] - (2*a^2*Cot[x])/Sqr
t[a + a*Csc[x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
```

```
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a+a\csc(x))^{3/2}\,dx &= -\frac{2a^2\cot(x)}{\sqrt{a+a\csc(x)}}+(2a)\int \frac{\frac{a}{2}+\frac{1}{2}a\csc(x)}{\sqrt{a+a\csc(x)}}\,dx\\ &= -\frac{2a^2\cot(x)}{\sqrt{a+a\csc(x)}}+a\int \sqrt{a+a\csc(x)}\,dx\\ &= -\frac{2a^2\cot(x)}{\sqrt{a+a\csc(x)}}-\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,\frac{a\cot(x)}{\sqrt{a+a\csc(x)}}\right)\\ &= -2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\cot(x)}{\sqrt{a+a\csc(x)}}\right)-\frac{2a^2\cot(x)}{\sqrt{a+a\csc(x)}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 69, normalized size = 1.57

$$-\frac{2a\left(\text{ArcTan}\left(\sqrt{-1+\csc(x)}\right)+\sqrt{-1+\csc(x)}\right)\sqrt{a(1+\csc(x))}\,\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{-1+\csc(x)}\,\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Csc[x])^(3/2),x]
[Out] (-2*a*(ArcTan[Sqrt[-1 + Csc[x]]] + Sqrt[-1 + Csc[x]])*Sqrt[a*(1 + Csc[x])]*
(Cos[x/2] - Sin[x/2]))/(Sqrt[-1 + Csc[x]]*(Cos[x/2] + Sin[x/2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(36) = 72.
time = 0.13, size = 275, normalized size = 6.25

method	result
default	$\frac{\left(-\sin(x)\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\ln\left(-\frac{\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\sqrt{2}\,\sin(x)+\sin(x)-\cos(x)+1}{\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\sqrt{2}\,\sin(x)-\sin(x)+\cos(x)-1}\right)-4\sin(x)\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\arctan\left(\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*csc(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-sin(x)*(-(cos(x)-1)/sin(x))^(1/2)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(
1/2)*sin(x)+sin(x)-cos(x)+1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin
(x)+cos(x)-1))-4*sin(x)*(-(cos(x)-1)/sin(x))^(1/2)*arctan((-(cos(x)-1)/sin(
x))^(1/2)*2^(1/2)+1)-4*sin(x)*(-(cos(x)-1)/sin(x))^(1/2)*arctan((-(cos(x)-1
)/sin(x))^(1/2)*2^(1/2)-1)-sin(x)*(-(cos(x)-1)/sin(x))^(1/2)*ln(-((-(cos(x)
-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1/
2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1))+2*sin(x)*2^(1/2)+2*cos(x)*2^(1/2)-2*2^(
1/2))*sin(x)*(a*(sin(x)+1)/sin(x))^(3/2)/(cos(x)*sin(x)+cos(x)^2-2*sin(x)+c
os(x)-2)*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(36) = 72$.
time = 0.48, size = 200, normalized size = 4.55

$$\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)\right)\right)a^{\frac{3}{2}}-\frac{1}{5}\sqrt{2}\left(a^{\frac{3}{2}}\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{5}{2}}+5a^{\frac{3}{2}}\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{3}{2}}+10a^{\frac{3}{2}}\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)-\frac{\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(x)}{\cos(x)+1}-\frac{15\sqrt{2}a^{\frac{3}{2}}\sin(x)^2}{(\cos(x)+1)^2}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(x)^3}{(\cos(x)+1)^3}-\frac{\sqrt{2}a^{\frac{3}{2}}\sin(x)^4}{(\cos(x)+1)^4}}{5\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(sin(x)/(cos(x) + 1)))
) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(sin(x)/(cos(x) + 1)))))*a
^(3/2) - 1/5*sqrt(2)*(a^(3/2)*(sin(x)/(cos(x) + 1))^(5/2) + 5*a^(3/2)*(sin(
x)/(cos(x) + 1))^(3/2) + 10*a^(3/2)*sqrt(sin(x)/(cos(x) + 1))) - 1/5*(5*sqr
t(2)*a^(3/2)*sin(x)/(cos(x) + 1) - 15*sqrt(2)*a^(3/2)*sin(x)^2/(cos(x) + 1)
^2 - 5*sqrt(2)*a^(3/2)*sin(x)^3/(cos(x) + 1)^3 - sqrt(2)*a^(3/2)*sin(x)^4/(
cos(x) + 1)^4)/(sin(x)/(cos(x) + 1))^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(36) = 72$.
time = 4.14, size = 212, normalized size = 4.82

$$\left[\frac{(a\cos(x)+a\sin(x)+a)\sqrt{-a}\log\left(\frac{2a\cos(x)^2-2\left(\cos(x)^2+(\cos(x)+1)\sin(x)-1\right)\sqrt{-a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}+a\cos(x)-(2a\cos(x)+a)\sin(x)-a}{\cos(x)+\sin(x)+1}\right)-2(a\cos(x)-a\sin(x)+a)\sqrt{\frac{a\sin(x)+a}{\sin(x)}}}{\cos(x)+\sin(x)+1},\ \frac{2\left((a\cos(x)+a\sin(x)+a)\sqrt{a}\arctan\left(-\frac{\sqrt{a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}(\cos(x)-\sin(x)+1)}{a\cos(x)+a\sin(x)+a}\right)-(a\cos(x)-a\sin(x)+a)\sqrt{\frac{a\sin(x)+a}{\sin(x)}}\right)}{\cos(x)+\sin(x)+1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(3/2),x, algorithm="fricas")
```

```
[Out] [((a*cos(x) + a*sin(x) + a)*sqrt(-a)*log((2*a*cos(x)^2 - 2*(cos(x)^2 + (cos
(x) + 1)*sin(x) - 1)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x)) + a*cos(x) - (2*a
*cos(x) + a)*sin(x) - a)/(cos(x) + sin(x) + 1)) - 2*(a*cos(x) - a*sin(x) +
a)*sqrt((a*sin(x) + a)/sin(x)))/(cos(x) + sin(x) + 1), 2*((a*cos(x) + a*sin
(x) + a)*sqrt(a)*arctan(-sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) - sin(
x) + 1)/(a*cos(x) + a*sin(x) + a)) - (a*cos(x) - a*sin(x) + a)*sqrt((a*sin(
x) + a)/sin(x)))/(cos(x) + sin(x) + 1)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\csc(x)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))**(3/2),x)
```

```
[Out] Integral((a*csc(x) + a)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(36) = 72.
time = 0.56, size = 195, normalized size = 4.43

$$\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\,a-\frac{\sqrt{2}\,a^{2}}{\sqrt{a\tan\left(\frac{1}{2}x\right)}}+\left(a\sqrt{|a|}+|a|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}+2\sqrt{a\tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)+\left(a\sqrt{|a|}+|a|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}-2\sqrt{a\tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)+\frac{1}{2}\left(a\sqrt{|a|}-|a|^{\frac{3}{2}}\right)\log\left(a\tan\left(\frac{1}{2}x\right)+\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\sqrt{|a|}+|a|\right)-\frac{1}{2}\left(a\sqrt{|a|}-|a|^{\frac{3}{2}}\right)\log\left(a\tan\left(\frac{1}{2}x\right)-\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\sqrt{|a|}+|a|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*sqrt(a*tan(1/2*x))*a - sqrt(2)*a^2/sqrt(a*tan(1/2*x)) + (a*sqrt(abs
(a)) + abs(a)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) + 2*sqrt(a*ta
n(1/2*x)))/sqrt(abs(a))) + (a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(-1/2*sqrt
(2)*(sqrt(2)*sqrt(abs(a)) - 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a))) + 1/2*(a*sq
rt(abs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x) + sqrt(2)*sqrt(a*tan(1/2*x))*sq
rt(abs(a)) + abs(a)) - 1/2*(a*sqrt(abs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x)
 - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+\frac{a}{\sin\left(x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(x))^(3/2),x)
```

```
[Out] int((a + a/sin(x))^(3/2), x)
```

3.15 $\int \sqrt{a+a\csc(x)}\,dx$

Optimal. Leaf size=26

$$-2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cot(x)}{\sqrt{a+a\csc(x)}}\right)$$

```
[Out] -2*arctan(cot(x)*a^(1/2)/(a+a*csc(x))^(1/2))*a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3859, 209}

$$-2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\cot(x)}{\sqrt{a\csc(x)+a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Csc[x]],x]
```

```
[Out] -2*Sqrt[a]*ArcTan[(Sqrt[a]*Cot[x])/Sqrt[a + a*Csc[x]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\csc(x)}\,dx &= -\left((2a)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\cot(x)}{\sqrt{a+a\csc(x)}}\right)\right)\\ &= -2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cot(x)}{\sqrt{a+a\csc(x)}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 32, normalized size = 1.23

$$-\frac{2a\text{ArcTan}\left(\sqrt{-1+\csc(x)}\right)\cot(x)}{\sqrt{-1+\csc(x)}\sqrt{a(1+\csc(x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Csc[x]],x]
```

```
[Out] (-2*a*ArcTan[Sqrt[-1 + Csc[x]]]*Cot[x])/(Sqrt[-1 + Csc[x]]*Sqrt[a*(1 + Csc[
x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(20) = 40.
time = 0.22, size = 199, normalized size = 7.65

method	result
default	$\frac{\sqrt{2}\,\sqrt{\frac{a(\sin(x)+1)}{\sin(x)}}\,\sin(x)\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\left(\ln\left(-\frac{\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\sqrt{2}\,\sin(x)+\sin(x)-\cos(x)+1}{\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\sqrt{2}\,\sin(x)-\sin(x)+\cos(x)-1}\right)+4\arctan\left(\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\right.\right.}{2-2\cos(x)+2\sin(x)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*csc(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(a*(sin(x)+1)/sin(x))^(1/2)*sin(x)*(-(cos(x)-1)/sin(x))^(1/2)*(
ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1)/((-(cos(x)-
1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))+4*arctan((-(cos(x)-1)/sin
(x))^(1/2)*2^(1/2)+1)+4*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)-1)+ln(-((
-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin
(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1)))/(1-cos(x)+sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(20) = 40.
time = 0.48, size = 148, normalized size = 5.69

$$-\frac{2}{3}\sqrt{2}\sqrt{a}\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{3}{2}}+\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)\right)\right)\sqrt{a}-2\sqrt{2}\sqrt{a}\sqrt{\frac{\sin(x)}{\cos(x)+1}}+\frac{2\left(\frac{3\sqrt{2}\sqrt{a}\sin(x)}{\cos(x)+1}+\frac{\sqrt{2}\sqrt{a}\sin(x)^2}{(\cos(x)+1)^2}\right)}{3\sqrt{\frac{\sin(x)}{\cos(x)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(2)*sqrt(a)*(sin(x)/(cos(x) + 1))^(3/2) + sqrt(2)*(sqrt(2)*arctan(
1/2*sqrt(2)*(sqrt(2) + 2*sqrt(sin(x)/(cos(x) + 1)))) + sqrt(2)*arctan(-1/2*
```

```
sqrt(2)*(sqrt(2) - 2*sqrt(sin(x)/(cos(x) + 1)))))*sqrt(a) - 2*sqrt(2)*sqrt(
a)*sqrt(sin(x)/(cos(x) + 1)) + 2/3*(3*sqrt(2)*sqrt(a)*sin(x)/(cos(x) + 1) +
 sqrt(2)*sqrt(a)*sin(x)^2/(cos(x) + 1)^2)/sqrt(sin(x)/(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(20) = 40.
time = 3.82, size = 120, normalized size = 4.62

$$\left[\sqrt{-a}\log\left(\frac{2\,a\cos\left(x\right)^{2}-2\left(\cos\left(x\right)^{2}+\left(\cos\left(x\right)+1\right)\sin\left(x\right)-1\right)\sqrt{-a}\sqrt{\frac{a\sin\left(x\right)+a}{\sin\left(x\right)}}+a\cos\left(x\right)-\left(2\,a\cos\left(x\right)+a\right)\sin\left(x\right)-a}{\cos\left(x\right)+\sin\left(x\right)+1}\right),\ 2\sqrt{a}\arctan\left(-\frac{\sqrt{a}\sqrt{\frac{a\sin\left(x\right)+a}{\sin\left(x\right)}}\left(\cos\left(x\right)-\sin\left(x\right)+1\right)}{a\cos\left(x\right)+a\sin\left(x\right)+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cos(x)^2 - 2*(cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt(-
a)*sqrt((a*sin(x) + a)/sin(x)) + a*cos(x) - (2*a*cos(x) + a)*sin(x) - a)/(c
os(x) + sin(x) + 1)), 2*sqrt(a)*arctan(-sqrt(a)*sqrt((a*sin(x) + a)/sin(x))
*(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\csc\left(x\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a*csc(x) + a), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(20) = 40.
time = 0.59, size = 353, normalized size = 13.58

$$\frac{1}{4}\sqrt{2}\left(\frac{2\sqrt{2}\left(a\sqrt{|a|}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^{3}+\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)+1\right)+|a|^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^{3}+\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)+1\right)\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}+2\sqrt{a\tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a}+\frac{2\sqrt{2}\left(a\sqrt{|a|}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^{3}+\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)+1\right)+|a|^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^{3}+\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)+1\right)\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}-2\sqrt{a\tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a}+\frac{\sqrt{2}\left(a\sqrt{|a|}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^{3}+\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)+1\right)-|a|^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^{3}+\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)+1\right)\right)\log\left(a\tan\left(\frac{1}{2}x\right)+\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\sqrt{|a|}+|a|\right)}{a}-\frac{\sqrt{2}\left(a\sqrt{|a|}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^{3}+\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)+1\right)-|a|^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^{3}+\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)+1\right)\right)\log\left(a\tan\left(\frac{1}{2}x\right)-\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\sqrt{|a|}+|a|\right)}{a}\right)\mathrm{sgn}\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*(a*sqrt(abs(a))*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + ta
n(1/2*x) + 1) + abs(a)^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) +
 1))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) + 2*sqrt(a*tan(1/2*x)))/sqrt(
abs(a)))/a + 2*sqrt(2)*(a*sqrt(abs(a))*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + ta
n(1/2*x) + 1) + abs(a)^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) +
 1))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) - 2*sqrt(a*tan(1/2*x)))/sqrt
(abs(a)))/a + sqrt(2)*(a*sqrt(abs(a))*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan
```

```
(1/2*x) + 1) - abs(a)^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) +
1))*log(a*tan(1/2*x) + sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))/a
- sqrt(2)*(a*sqrt(abs(a))*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) + 1)
 - abs(a)^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) + 1))*log(a*ta
n(1/2*x) - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))/a)*sgn(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a+\frac{a}{\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(x))^(1/2),x)
```

```
[Out] int((a + a/sin(x))^(1/2), x)
```

3.16 $\int \frac{1}{\sqrt{a+a\csc(x)}}\,dx$

Optimal. Leaf size=62

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{a+a\csc(x)}}\right)}{\sqrt{a}}+\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{2}\sqrt{a+a\csc(x)}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctan(cot(x)*a^(1/2)/(a+a*csc(x))^(1/2))/a^(1/2)+arctan(1/2*cot(x)*a^(1
/2)*2^(1/2)/(a+a*csc(x))^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3861, 3859, 209, 3880}

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{2}\sqrt{a\csc(x)+a}}\right)}{\sqrt{a}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{a\csc(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Csc[x]],x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Cot[x])/Sqrt[a + a*Csc[x]]])/Sqrt[a] + (Sqrt[2]*ArcTan[
(Sqrt[a]*Cot[x])/(Sqrt[2]*Sqrt[a + a*Csc[x]])])/Sqrt[a]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\csc(x)}}\,dx = \frac{\int \sqrt{a+a\csc(x)}\,dx}{a} - \int \frac{\csc(x)}{\sqrt{a+a\csc(x)}}\,dx$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\cot(x)}{\sqrt{a+a\csc(x)}}\right)\right) + 2\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{a\,\mathrm{c}}{\sqrt{a+}}\right.$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\cot(x)}{\sqrt{a+a\csc(x)}}\right)}{\sqrt{a}} + \frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\cot(x)}{\sqrt{2}\,\sqrt{a+a\csc(x)}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.14, size = 54, normalized size = 0.87

$$\frac{\left(-2\text{ArcTan}\left(\sqrt{-1+\csc(x)}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\csc(x)}}{\sqrt{2}}\right)\right)\cot(x)}{\sqrt{-1+\csc(x)}\,\sqrt{a(1+\csc(x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Csc[x]],x]
[Out] ((-2*ArcTan[Sqrt[-1 + Csc[x]]] + Sqrt[2]*ArcTan[Sqrt[-1 + Csc[x]]/Sqrt[2]])
*Cot[x])/(Sqrt[-1 + Csc[x]]*Sqrt[a*(1 + Csc[x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(47) = 94.
time = 0.12, size = 221, normalized size = 3.56

method	result
default	$-\frac{\left(4\sqrt{2}\,\arctan\left(\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\right)-\ln\left(-\frac{\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\sqrt{2}\,\sin(x)+\sin(x)-\cos(x)+1}{\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\sqrt{2}\,\sin(x)-\sin(x)+\cos(x)-1}\right)-4\arctan\left(\sqrt{-\frac{\cos(x)-1}{\sin(x)}}\,\sqrt{2}+1\right.\right.}{4\sqrt{\frac{a(\sin(x)+1)}{\sin(x)}}\,\sin(x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*csc(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(4*2^(1/2)*arctan((-(cos(x)-1)/sin(x))^(1/2))-ln(-((-(cos(x)-1)/sin(x)
)^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)
*sin(x)-sin(x)+cos(x)-1))-4*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)-4*
arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)-1)-ln(-((-(cos(x)-1)/sin(x))^(1/2
)*2^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x
)+sin(x)-cos(x)+1)))*(1-cos(x)+sin(x))/(a*(sin(x)+1)/sin(x))^(1/2)/sin(x)/(
-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 83, normalized size = 1.34

$$\frac{\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)\right)+\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)\right)\right)}{\sqrt{a}}-\frac{2\sqrt{2}\arctan\left(\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(sin(x)/(cos(x) + 1)))
) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(sin(x)/(cos(x) + 1)))))/s
qrt(a) - 2*sqrt(2)*arctan(sqrt(sin(x)/(cos(x) + 1)))/sqrt(a)
```

Fricas [A]
time = 2.50, size = 219, normalized size = 3.53

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\log\left(\frac{\sqrt{2}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}\sqrt{-\frac{1}{a}}\sin(x)+\cos(x)}{\sin(x)+1}\right)-\sqrt{-a}\log\left(\frac{2\,a\cos(x)^2+2\left(\cos(x)^2+(\cos(x)+1)\sin(x)-1\right)\sqrt{-a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}+a\cos(x)-(2\,a\cos(x)+a)\sin(x)-a}{\cos(x)+\sin(x)+1}\right)}{a},\ -\frac{2\left(\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}\sin(x)}{\sqrt{a}(\cos(x)+\sin(x)+1)}\right)-\sqrt{a}\arctan\left(-\frac{\sqrt{a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}(\cos(x)-\sin(x)+1)}{a\cos(x)+a\sin(x)+a}\right)\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a*sqrt(-1/a)*log((sqrt(2)*sqrt((a*sin(x) + a)/sin(x))*sqrt(-1/a)*
sin(x) + cos(x))/(sin(x) + 1)) - sqrt(-a)*log((2*a*cos(x)^2 + 2*(cos(x)^2 +
 (cos(x) + 1)*sin(x) - 1)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x)) + a*cos(x) -
 (2*a*cos(x) + a)*sin(x) - a)/(cos(x) + sin(x) + 1)))/a, -2*(sqrt(2)*sqrt(a
)*arctan(sqrt(2)*sqrt((a*sin(x) + a)/sin(x))*sin(x)/(sqrt(a)*(cos(x) + sin(
x) + 1))) - sqrt(a)*arctan(-sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) - s
in(x) + 1)/(a*cos(x) + a*sin(x) + a)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\csc(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(x))**(1/2),x)

[Out] Integral(1/sqrt(a*csc(x) + a), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(47) = 94.
time = 0.57, size = 205, normalized size = 3.31

$$-\frac{4\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}x\right)}}{\sqrt{a}}\right)-\frac{2\left(a\sqrt{|a|}+|a|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}+2\sqrt{a\tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a}-\frac{2\left(a\sqrt{|a|}+|a|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}-2\sqrt{a\tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a}-\frac{\left(a\sqrt{|a|}-|a|^{\frac{3}{2}}\right)\log\left(a\tan\left(\frac{1}{2}x\right)+\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\sqrt{|a|}+|a|\right)}{a}+\frac{\left(a\sqrt{|a|}-|a|^{\frac{3}{2}}\right)\log\left(a\tan\left(\frac{1}{2}x\right)-\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\sqrt{|a|}+|a|\right)}{a}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(x))^(1/2),x, algorithm="giac")

[Out] -1/2*(4*sqrt(2)*sqrt(a)*arctan(sqrt(a*tan(1/2*x))/sqrt(a)) - 2*(a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) + 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a)))/a - 2*(a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) - 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a)))/a - (a*sqrt(abs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x) + sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))/a + (a*sqrt(abs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x) - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))/a)/a

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a+\frac{a}{\sin\left(x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + a/sin(x))^(1/2),x)

[Out] int(1/(a + a/sin(x))^(1/2), x)

3.17 $\int \frac{1}{(a+a\csc(x))^{3/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{a+a\csc(x)}}\right)}{a^{3/2}}+\frac{5\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{2}\sqrt{a+a\csc(x)}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{\cot(x)}{2(a+a\csc(x))^{3/2}}$$

```
[Out] -2*arctan(cot(x)*a^(1/2)/(a+a*csc(x))^(1/2))/a^(3/2)+1/2*cot(x)/(a+a*csc(x)
)^(3/2)+5/4*arctan(1/2*cot(x)*a^(1/2)*2^(1/2)/(a+a*csc(x))^(1/2))/a^(3/2)*2
^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3862, 4005, 3859, 209, 3880}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{a\csc(x)+a}}\right)}{a^{3/2}}+\frac{5\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{2}\sqrt{a\csc(x)+a}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{\cot(x)}{2(a\csc(x)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Csc[x])^(-3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Cot[x])/Sqrt[a + a*Csc[x]]])/a^(3/2) + (5*ArcTan[(Sqrt[
a]*Cot[x])/(Sqrt[2]*Sqrt[a + a*Csc[x]])])/(2*Sqrt[2]*a^(3/2)) + Cot[x]/(2*(
a + a*Csc[x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
```

```
gerQ[2*n]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\csc(x))^{3/2}}\,dx &= \frac{\cot(x)}{2(a+a\csc(x))^{3/2}} - \frac{\int \frac{-2a+\frac{1}{2}a\csc(x)}{\sqrt{a+a\csc(x)}}\,dx}{2a^2}\\
&= \frac{\cot(x)}{2(a+a\csc(x))^{3/2}} + \frac{\int \sqrt{a+a\csc(x)}\,dx}{a^2} - \frac{5\int \frac{\csc(x)}{\sqrt{a+a\csc(x)}}\,dx}{4a}\\
&= \frac{\cot(x)}{2(a+a\csc(x))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\cot(x)}{\sqrt{a+a\csc(x)}}\right)}{a} + \frac{5\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx\right.}{}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{a+a\csc(x)}}\right)}{a^{3/2}} + \frac{5\tan^{-1}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{2}\sqrt{a+a\csc(x)}}\right)}{2\sqrt{2}\,a^{3/2}} + \frac{\cot(x)}{2(a+a\csc(x))^{3}}
\end{aligned}$$

Mathematica [A]
time = 0.44, size = 129, normalized size = 1.59

$$-\frac{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\left(2-2\csc(x)+8\text{ArcTan}\left(\sqrt{-1+\csc(x)}\right)\sqrt{-1+\csc(x)}\,(1+\csc(x))-5\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\csc(x)}}{\sqrt{2}}\right)\sqrt{-1+\csc(x)}\,\csc(x)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2\right)}{4a(-1+\csc(x))\sqrt{a(1+\csc(x))}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Csc[x])^(-3/2),x]
```

```
[Out] -1/4*((Cos[x/2] - Sin[x/2])*(2 - 2*Csc[x] + 8*ArcTan[Sqrt[-1 + Csc[x]]]*Sqr
t[-1 + Csc[x]]*(1 + Csc[x]) - 5*Sqrt[2]*ArcTan[Sqrt[-1 + Csc[x]]/Sqrt[2]]*S
```

```
qrt[-1 + Csc[x]]*Csc[x]*(Cos[x/2] + Sin[x/2])^2))/(a*(-1 + Csc[x])*Sqrt[a*(
1 + Csc[x])]*(Cos[x/2] + Sin[x/2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1140 vs. $2(60) = 120$.
time = 0.13, size = 1141, normalized size = 14.09

method	result	size
default	Expression too large to display	1141

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*csc(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(cos(x)-1)*(-(-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-cos(x)^2*2^(1/2)
*(-(cos(x)-1)/sin(x))^(3/2)+sin(x)*2^(1/2)*(-(cos(x)-1)/sin(x))^(3/2)+cos(x
)^2*2^(1/2)*(-(cos(x)-1)/sin(x))^(1/2)-16*arctan((-(cos(x)-1)/sin(x))^(1/2)
*2^(1/2)-1)-16*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)-4*ln(-((-(cos(x
)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1
/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1))-4*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1
/2)*sin(x)+sin(x)-cos(x)+1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(
x)+cos(x)-1))+2^(1/2)*(-(cos(x)-1)/sin(x))^(3/2)+2*sin(x)*cos(x)*ln(-((-(co
s(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))
^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1))-10*cos(x)^2*2^(1/2)*arctan((-(cos(x
)-1)/sin(x))^(1/2))+20*sin(x)*2^(1/2)*arctan((-(cos(x)-1)/sin(x))^(1/2))-10
*cos(x)*2^(1/2)*arctan((-(cos(x)-1)/sin(x))^(1/2))+2*sin(x)*cos(x)*ln(-((-(
cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1)/((-(cos(x)-1)/sin(x
))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))+8*sin(x)*cos(x)*arctan((-(cos(x)-
1)/sin(x))^(1/2)*2^(1/2)+1)+8*sin(x)*cos(x)*arctan((-(cos(x)-1)/sin(x))^(1/
2)*2^(1/2)-1)-10*sin(x)*cos(x)*2^(1/2)*arctan((-(cos(x)-1)/sin(x))^(1/2))+2
0*2^(1/2)*arctan((-(cos(x)-1)/sin(x))^(1/2))-(-(cos(x)-1)/sin(x))^(1/2)*2^(
1/2)+2*cos(x)^2*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x
)+1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))+8*cos(x)^
2*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)+8*cos(x)^2*arctan((-(cos(x)-
1)/sin(x))^(1/2)*2^(1/2)-1)+2*cos(x)^2*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1
/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(
x)-cos(x)+1))-4*sin(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x
)-cos(x)+1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))-16
*sin(x)*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)-16*sin(x)*arctan((-(co
s(x)-1)/sin(x))^(1/2)*2^(1/2)-1)-4*sin(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2
^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+s
in(x)-cos(x)+1))+2*cos(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+si
n(x)-cos(x)+1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))
+8*cos(x)*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)+8*cos(x)*arctan((-(c
os(x)-1)/sin(x))^(1/2)*2^(1/2)-1)+2*cos(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*
2^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+
```

```
sin(x)-cos(x)+1))+sin(x)*cos(x)*2^(1/2)*(-(cos(x)-1)/sin(x))^(3/2)-sin(x)*c
os(x)*2^(1/2)*(-(cos(x)-1)/sin(x))^(1/2))/(a*(sin(x)+1)/sin(x))^(3/2)/sin(x
)^3/(-(cos(x)-1)/sin(x))^(3/2)*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(60) = 120$.

time = 0.48, size = 150, normalized size = 1.85

$$-\frac{\sqrt{2}\left(\frac{\sin(x)}{\cos(x)+1}\right)^{\frac{3}{2}} - \sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)+1}}}{2\left(a^{\frac{3}{2}} + \frac{2a^{\frac{3}{2}}\sin(x)}{\cos(x)+1} + \frac{a^{\frac{3}{2}}\sin(x)^2}{(\cos(x)+1)^2}\right)} + \frac{\sqrt{2}\left(\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)\right) + \sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)\right)\right)}{a^{\frac{3}{2}}} - \frac{5\sqrt{2}\arctan\left(\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*(sin(x)/(cos(x) + 1))^(3/2) - sqrt(2)*sqrt(sin(x)/(cos(x) + 1
)))/(a^(3/2) + 2*a^(3/2)*sin(x)/(cos(x) + 1) + a^(3/2)*sin(x)^2/(cos(x) + 1
)^2) + sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(sin(x)/(cos(x)
 + 1)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(sin(x)/(cos(x) + 1
)))))/a^(3/2) - 5/2*sqrt(2)*arctan(sqrt(sin(x)/(cos(x) + 1)))/a^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(60) = 120$.

time = 4.55, size = 427, normalized size = 5.27

$$\left[-\frac{5\sqrt{2}\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)\sqrt{-a}\log\left(-\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}\sin(x)-a\cos(x)}{\sin(x)+1}\right) + 4\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)\sqrt{-a}\log\left(\frac{2a\cos(x)^2+2\left(\cos(x)^2+(\cos(x)+1)\sin(x)-1\right)\sqrt{-a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}+a\cos(x)-(2a\cos(x)+a)\sin(x)-a}{\cos(x)+\sin(x)+1}\right) + 2\left(\cos(x)^2+(\cos(x)+1)\sin(x)-1\right)\sqrt{\frac{a\sin(x)+a}{\sin(x)}}}{4\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-(a^2\cos(x)+2a^2)\sin(x)\right)}, \frac{5\sqrt{2}\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}(\cos(x)+1)}{a\cos(x)+a\sin(x)+a}\right) + 4\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)\sqrt{a}\arctan\left(-\frac{\sqrt{a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}(\cos(x)-\sin(x)+1)}{a\cos(x)+a\sin(x)+a}\right) - \left(\cos(x)^2+(\cos(x)+1)\sin(x)-1\right)\sqrt{\frac{a\sin(x)+a}{\sin(x)}}}{2\left(a^2\cos(x)^2-a^2\cos(x)-2a^2-(a^2\cos(x)+2a^2)\sin(x)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(5*sqrt(2)*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*sqrt(-a)*log
(-(sqrt(2)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x))*sin(x) - a*cos(x))/(sin(x)
+ 1)) + 4*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*sqrt(-a)*log((2*a*c
os(x)^2 + 2*(cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt(-a)*sqrt((a*sin(x) +
a)/sin(x)) + a*cos(x) - (2*a*cos(x) + a)*sin(x) - a)/(cos(x) + sin(x) + 1))
 + 2*(cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt((a*sin(x) + a)/sin(x)))/(a^2
*cos(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x)), 1/2*(5*sqrt(
2)*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*sqrt(a)*arctan(sqrt(2)*sqr
t(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) + 1)/(a*cos(x) + a*sin(x) + a)) +
4*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*sqrt(a)*arctan(-sqrt(a)*sqr
t((a*sin(x) + a)/sin(x))*(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)) -
 (cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt((a*sin(x) + a)/sin(x)))/(a^2*cos
(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\csc(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))**(3/2),x)
```

```
[Out] Integral((a*csc(x) + a)**(-3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(60) = 120.
time = 0.47, size = 243, normalized size = 3.00

$$-\frac{5\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}x\right)}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}+\frac{\left(a\sqrt{|a|}+|a|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}+2\sqrt{a\tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a^3}+\frac{\left(a\sqrt{|a|}+|a|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}-2\sqrt{a\tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a^3}+\frac{\left(a\sqrt{|a|}-|a|^{\frac{3}{2}}\right)\log\left(a\tan\left(\frac{1}{2}x\right)+\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\sqrt{|a|}+|a|\right)}{2a^3}-\frac{\left(a\sqrt{|a|}-|a|^{\frac{3}{2}}\right)\log\left(a\tan\left(\frac{1}{2}x\right)-\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\sqrt{|a|}+|a|\right)}{2a^3}-\frac{\sqrt{2}\left(\sqrt{a\tan\left(\frac{1}{2}x\right)}a\tan\left(\frac{1}{2}x\right)-\sqrt{a\tan\left(\frac{1}{2}x\right)}a\right)}{2\left(a\tan\left(\frac{1}{2}x\right)+a\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(3/2),x, algorithm="giac")
```

```
[Out] -5/2*sqrt(2)*arctan(sqrt(a*tan(1/2*x))/sqrt(a))/a^(3/2) + (a*sqrt(abs(a)) +
 abs(a)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) + 2*sqrt(a*tan(1/2*
x)))/sqrt(abs(a)))/a^3 + (a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(-1/2*sqrt(2
)*(sqrt(2)*sqrt(abs(a)) - 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a)))/a^3 + 1/2*(a*
sqrt(abs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x) + sqrt(2)*sqrt(a*tan(1/2*x))*
sqrt(abs(a)) + abs(a))/a^3 - 1/2*(a*sqrt(abs(a)) - abs(a)^(3/2))*log(a*tan(
1/2*x) - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))/a^3 - 1/2*sqrt(2
)*(sqrt(a*tan(1/2*x))*a*tan(1/2*x) - sqrt(a*tan(1/2*x))*a)/((a*tan(1/2*x) +
 a)^2*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{a}{\sin(x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/sin(x))^(3/2),x)
```

```
[Out] int(1/(a + a/sin(x))^(3/2), x)
```

3.18 $\int \frac{1}{(a+a\csc(x))^{5/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{a+a\csc(x)}}\right)}{a^{5/2}}+\frac{43\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{2}\sqrt{a+a\csc(x)}}\right)}{16\sqrt{2}a^{5/2}}+\frac{\cot(x)}{4(a+a\csc(x))^{5/2}}+\frac{11\cot(x)}{16a(a+a\csc(x))^{3/2}}$$

```
[Out] -2*arctan(cot(x)*a^(1/2)/(a+a*csc(x))^(1/2))/a^(5/2)+1/4*cot(x)/(a+a*csc(x)
)^(5/2)+11/16*cot(x)/a/(a+a*csc(x))^(3/2)+43/32*arctan(1/2*cot(x)*a^(1/2)*2
^(1/2)/(a+a*csc(x))^(1/2))/a^(5/2)*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3862, 4007, 4005, 3859, 209, 3880}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{a\csc(x)+a}}\right)}{a^{5/2}}+\frac{43\text{ArcTan}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{2}\sqrt{a\csc(x)+a}}\right)}{16\sqrt{2}a^{5/2}}+\frac{11\cot(x)}{16a(a\csc(x)+a)^{3/2}}+\frac{\cot(x)}{4(a\csc(x)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Csc[x])^(-5/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Cot[x])/Sqrt[a + a*Csc[x]]])/a^(5/2) + (43*ArcTan[(Sqrt
[a]*Cot[x])/(Sqrt[2]*Sqrt[a + a*Csc[x]])])/(16*Sqrt[2]*a^(5/2)) + Cot[x]/(4
*(a + a*Csc[x])^(5/2)) + (11*Cot[x])/(16*a*(a + a*Csc[x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
```

```
gerQ[2*n]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\csc(x))^{5/2}}\,dx &= \frac{\cot(x)}{4(a+a\csc(x))^{5/2}} - \frac{\int \frac{-4a+\frac{3}{2}a\csc(x)}{(a+a\csc(x))^{3/2}}\,dx}{4a^2} \\
&= \frac{\cot(x)}{4(a+a\csc(x))^{5/2}} + \frac{11\cot(x)}{16a(a+a\csc(x))^{3/2}} + \frac{\int \frac{8a^2-\frac{11}{4}a^2\csc(x)}{\sqrt{a+a\csc(x)}}\,dx}{8a^4} \\
&= \frac{\cot(x)}{4(a+a\csc(x))^{5/2}} + \frac{11\cot(x)}{16a(a+a\csc(x))^{3/2}} + \frac{\int \sqrt{a+a\csc(x)}\,dx}{a^3} - \frac{43\int \frac{\csc(}{\sqrt{a+a}}}{32a} \\
&= \frac{\cot(x)}{4(a+a\csc(x))^{5/2}} + \frac{11\cot(x)}{16a(a+a\csc(x))^{3/2}} - \frac{2\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\cot(x)}{\sqrt{a+a\csc(x)}}\right.}{a^2} \\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{a+a\csc(x)}}\right)}{a^{5/2}} + \frac{43\tan^{-1}\left(\frac{\sqrt{a}\cot(x)}{\sqrt{2}\sqrt{a+a\csc(x)}}\right)}{16\sqrt{2}\,a^{5/2}} + \frac{\cot(x)}{4(a+a\csc(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.54, size = 139, normalized size = 1.39

$$\frac{\csc^2(x)\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\left(7+15\cos(2x)-64\text{ArcTan}\left(\sqrt{-1+\csc(x)}\right)\sqrt{-1+\csc(x)}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4+43\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\csc(x)}}{\sqrt{2}}\right)\sqrt{-1+\csc(x)}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4+8\sin(x)\right)}{32(a(1+\csc(x)))^{5/2}\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Csc[x])^(-5/2),x]
```

```
[Out] (Csc[x]^2*(Cos[x/2] + Sin[x/2])*(7 + 15*Cos[2*x] - 64*ArcTan[Sqrt[-1 + Csc[
x]]]*Sqrt[-1 + Csc[x]]*(Cos[x/2] + Sin[x/2])^4 + 43*Sqrt[2]*ArcTan[Sqrt[-1
+ Csc[x]]/Sqrt[2]]*Sqrt[-1 + Csc[x]]*(Cos[x/2] + Sin[x/2])^4 + 8*Sin[x]))/(
32*(a*(1 + Csc[x]))^(5/2)*(Cos[x/2] - Sin[x/2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1960 vs. $2(75) = 150$.
time = 0.12, size = 1961, normalized size = 19.61

method	result	size
default	Expression too large to display	1961

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*csc(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(cos(x)-1)^2*(-11*(-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+19*cos(x
)^2*2^(1/2)*(-(cos(x)-1)/sin(x))^(3/2)-19*sin(x)*2^(1/2)*(-(cos(x)-1)/sin(x
))^(3/2)+11*cos(x)^2*2^(1/2)*(-(cos(x)-1)/sin(x))^(1/2)-128*cos(x)^3*arctan
((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)-1)-32*cos(x)^3*ln(-((-(cos(x)-1)/sin(x)
)^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)
*sin(x)+sin(x)-cos(x)+1))-32*cos(x)^3*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/
2)*sin(x)+sin(x)-cos(x)+1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x
)+cos(x)-1))-128*cos(x)^3*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)-11*c
os(x)*2^(1/2)*(-(cos(x)-1)/sin(x))^(1/2)+19*sin(x)*2^(1/2)*(-(cos(x)-1)/sin
(x))^(5/2)+19*cos(x)^3*2^(1/2)*(-(cos(x)-1)/sin(x))^(3/2)+19*cos(x)*2^(1/2)
*(-(cos(x)-1)/sin(x))^(5/2)+11*cos(x)^3*2^(1/2)*(-(cos(x)-1)/sin(x))^(1/2)-
19*cos(x)*2^(1/2)*(-(cos(x)-1)/sin(x))^(3/2)-11*cos(x)^3*2^(1/2)*(-(cos(x)-
1)/sin(x))^(7/2)-11*cos(x)^2*2^(1/2)*(-(cos(x)-1)/sin(x))^(7/2)+11*sin(x)*2
^(1/2)*(-(cos(x)-1)/sin(x))^(7/2)-19*cos(x)^3*2^(1/2)*(-(cos(x)-1)/sin(x))^
(5/2)+11*cos(x)*2^(1/2)*(-(cos(x)-1)/sin(x))^(7/2)-19*cos(x)^2*2^(1/2)*(-(c
os(x)-1)/sin(x))^(5/2)-172*sin(x)*cos(x)^2*2^(1/2)*arctan((-(cos(x)-1)/sin(
x))^(1/2))-512*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)-1)-512*arctan((-(c
os(x)-1)/sin(x))^(1/2)*2^(1/2)+1)-128*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/
2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x
)-cos(x)+1))-128*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(
x)+1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))+128*sin(
```

```
x)*cos(x)^2*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)-1)+32*sin(x)*cos(x)^2
*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)
-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1))+32*sin(x)*cos(x)^2*ln(-(
(-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1)/((-(cos(x)-1)/si
n(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))+128*sin(x)*cos(x)^2*arctan((-(
cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)+172*cos(x)^3*2^(1/2)*arctan((-(cos(x)-1)
/sin(x))^(1/2))-19*2^(1/2)*(-(cos(x)-1)/sin(x))^(3/2)+64*sin(x)*cos(x)*ln(-
((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/s
in(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1))-516*cos(x)^2*2^(1/2)*arctan((
-(cos(x)-1)/sin(x))^(1/2))+688*sin(x)*2^(1/2)*arctan((-(cos(x)-1)/sin(x))^(
1/2))-344*cos(x)*2^(1/2)*arctan((-(cos(x)-1)/sin(x))^(1/2))+64*sin(x)*cos(x
)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1)/((-(cos(x
)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))+256*sin(x)*cos(x)*arcta
n((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)+256*sin(x)*cos(x)*arctan((-(cos(x)-
1)/sin(x))^(1/2)*2^(1/2)-1)+11*2^(1/2)*(-(cos(x)-1)/sin(x))^(7/2)+19*2^(1/2
)*(-(cos(x)-1)/sin(x))^(5/2)-344*sin(x)*cos(x)*2^(1/2)*arctan((-(cos(x)-1)/
sin(x))^(1/2))+688*2^(1/2)*arctan((-(cos(x)-1)/sin(x))^(1/2))-11*(-(cos(x)-
1)/sin(x))^(1/2)*2^(1/2)+22*sin(x)*cos(x)*2^(1/2)*(-(cos(x)-1)/sin(x))^(7/2
)+38*sin(x)*cos(x)*2^(1/2)*(-(cos(x)-1)/sin(x))^(5/2)+96*cos(x)^2*ln(-((-(c
os(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1)/((-(cos(x)-1)/sin(x)
)^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))+384*cos(x)^2*arctan((-(cos(x)-1)/s
in(x))^(1/2)*2^(1/2)+1)+384*cos(x)^2*arctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1
/2)-1)+96*cos(x)^2*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+co
s(x)-1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1))-128*si
n(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1)/((-(co
s(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))-512*sin(x)*arctan((-
(cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)-512*sin(x)*arctan((-(cos(x)-1)/sin(x))^
(1/2)*2^(1/2)-1)-128*sin(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-
sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1
))+64*cos(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-cos(x)+1
)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)-sin(x)+cos(x)-1))+256*cos(x)*a
rctan((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)+1)+256*cos(x)*arctan((-(cos(x)-1)/
sin(x))^(1/2)*2^(1/2)-1)+64*cos(x)*ln(-((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*
sin(x)-sin(x)+cos(x)-1)/((-(cos(x)-1)/sin(x))^(1/2)*2^(1/2)*sin(x)+sin(x)-c
os(x)+1))+11*sin(x)*cos(x)^2*2^(1/2)*(-(cos(x)-1)/sin(x))^(7/2)+19*sin(x)*c
os(x)^2*2^(1/2)*(-(cos(x)-1)/sin(x))^(5/2)-19*sin(x)*cos(x)^2*2^(1/2)*(-(co
s(x)-1)/sin(x))^(3/2)-11*sin(x)*cos(x)^2*2^(1/2)*(-(cos(x)-1)/sin(x))^(1/2)
-38*sin(x)*cos(x)*2^(1/2)*(-(cos(x)-1)/sin(x))^(3/2)-22*sin(x)*cos(x)*2^(1/
2)*(-(cos(x)-1)/sin(x))^(1/2))/(a*(sin(x)+1)/sin(x))^(5/2)/sin(x)^5/(-(cos(
x)-1)/sin(x))^(5/2)*2^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*csc(x) + a)^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(75) = 150$.
time = 3.27, size = 546, normalized size = 5.46

$$\left[-\frac{43\sqrt{2}\left(\cos(x)^3+3\cos(x)^2+\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4\right)\sqrt{-a}\log\left(-\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}\sin(x)-a\cos(x)}{\sin(x)+1}\right)+32\left(\cos(x)^3+3\cos(x)^2+\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4\right)\sqrt{-a}\log\left(\frac{2a\cos(x)^2+2\left(\cos(x)^2+(\cos(x)+1)\sin(x)-1\right)\sqrt{-a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}+a\cos(x)-(2a\cos(x)+a)\sin(x)-a}{\cos(x)+\sin(x)+1}\right)-2\left(15\cos(x)^3+4\cos(x)^2-\left(15\cos(x)^2+11\cos(x)-4\right)\sin(x)-15\cos(x)-4\right)\sqrt{\frac{a\sin(x)+a}{\sin(x)}}}{32\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}, \frac{43\sqrt{2}\left(\cos(x)^3+3\cos(x)^2+\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}(\cos(x)+1)}{a\cos(x)+a\sin(x)+a}\right)+32\left(\cos(x)^3+3\cos(x)^2+\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4\right)\sqrt{a}\arctan\left(-\frac{\sqrt{a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}(\cos(x)-\sin(x)+1)}{a\cos(x)+a\sin(x)+a}\right)+\left(15\cos(x)^3+4\cos(x)^2-\left(15\cos(x)^2+11\cos(x)-4\right)\sin(x)-15\cos(x)-4\right)\sqrt{\frac{a\sin(x)+a}{\sin(x)}}}{16\left(a^3\cos(x)^3+3a^3\cos(x)^2-2a^3\cos(x)-4a^3+\left(a^3\cos(x)^2-2a^3\cos(x)-4a^3\right)\sin(x)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(43*sqrt(2)*(cos(x)^3 + 3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x
) - 2*cos(x) - 4)*sqrt(-a)*log(-(sqrt(2)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x
))*sin(x) - a*cos(x))/(sin(x) + 1)) + 32*(cos(x)^3 + 3*cos(x)^2 + (cos(x)^2
 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*sqrt(-a)*log((2*a*cos(x)^2 + 2*(cos
(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x)) + a*c
os(x) - (2*a*cos(x) + a)*sin(x) - a)/(cos(x) + sin(x) + 1)) - 2*(15*cos(x)^
3 + 4*cos(x)^2 - (15*cos(x)^2 + 11*cos(x) - 4)*sin(x) - 15*cos(x) - 4)*sqrt
((a*sin(x) + a)/sin(x)))/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*
a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x)), 1/16*(43*sqrt(2)*(cos(
x)^3 + 3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*sqrt(a
)*arctan(sqrt(2)*sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) + 1)/(a*cos(x)
 + a*sin(x) + a)) + 32*(cos(x)^3 + 3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*s
in(x) - 2*cos(x) - 4)*sqrt(a)*arctan(-sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(
cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)) + (15*cos(x)^3 + 4*cos(x)^2
 - (15*cos(x)^2 + 11*cos(x) - 4)*sin(x) - 15*cos(x) - 4)*sqrt((a*sin(x) + a
)/sin(x)))/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos
(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\csc(x)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))**(5/2),x)
```

```
[Out] Integral((a*csc(x) + a)**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(75) = 150$.
time = 0.49, size = 286, normalized size = 2.86

$$-\frac{43\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}x\right)}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}+\frac{\left(a\sqrt{|a|}+|a|^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}+2\sqrt{a\tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a^4}+\frac{\left(a\sqrt{|a|}+|a|^{\frac{3}{2}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|}-2\sqrt{a\tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a^4}+\frac{\left(a\sqrt{|a|}-|a|^{\frac{3}{2}}\right)\log\left(a\tan\left(\frac{1}{2}x\right)+\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\sqrt{|a|}+|a|\right)}{2a^4}-\frac{\left(a\sqrt{|a|}-|a|^{\frac{3}{2}}\right)\log\left(a\tan\left(\frac{1}{2}x\right)-\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}x\right)}\sqrt{|a|}+|a|\right)}{2a^4}-\frac{\sqrt{2}\left(11\sqrt{a\tan\left(\frac{1}{2}x\right)}a^3\tan\left(\frac{1}{2}x\right)^3+19\sqrt{a\tan\left(\frac{1}{2}x\right)}a^3\tan\left(\frac{1}{2}x\right)^2-19\sqrt{a\tan\left(\frac{1}{2}x\right)}a^3\tan\left(\frac{1}{2}x\right)-11\sqrt{a\tan\left(\frac{1}{2}x\right)}a^3\right)}{16\left(a\tan\left(\frac{1}{2}x\right)+a\right)^4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(5/2),x, algorithm="giac")
```

```
[Out] -43/16*sqrt(2)*arctan(sqrt(a*tan(1/2*x))/sqrt(a))/a^(5/2) + (a*sqrt(abs(a))
 + abs(a)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) + 2*sqrt(a*tan(1/
2*x)))/sqrt(abs(a)))/a^4 + (a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(-1/2*sqrt
(2)*(sqrt(2)*sqrt(abs(a)) - 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a)))/a^4 + 1/2*(
a*sqrt(abs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x) + sqrt(2)*sqrt(a*tan(1/2*x)
)*sqrt(abs(a)) + abs(a))/a^4 - 1/2*(a*sqrt(abs(a)) - abs(a)^(3/2))*log(a*ta
n(1/2*x) - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))/a^4 - 1/16*sqr
t(2)*(11*sqrt(a*tan(1/2*x))*a^3*tan(1/2*x)^3 + 19*sqrt(a*tan(1/2*x))*a^3*ta
n(1/2*x)^2 - 19*sqrt(a*tan(1/2*x))*a^3*tan(1/2*x) - 11*sqrt(a*tan(1/2*x))*a
^3)/((a*tan(1/2*x) + a)^4*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{a}{\sin(x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/sin(x))^(5/2),x)
```

```
[Out] int(1/(a + a/sin(x))^(5/2), x)
```

3.19 $\int \sqrt{\csc(e+fx)}\,\sqrt{a+a\csc(e+fx)}\,dx$

Optimal. Leaf size=37

$$-\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\cot(e+fx)}{\sqrt{a+a\csc(e+fx)}}\right)}{f}$$

[Out] -2*arcsinh(cot(f*x+e)*a^(1/2)/(a+a*csc(f*x+e))^(1/2))*a^(1/2)/f

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3886, 221}

$$-\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\cot(e+fx)}{\sqrt{a\csc(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Csc[e + f*x]]*Sqrt[a + a*Csc[e + f*x]],x]

[Out] (-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Cot[e + f*x])/Sqrt[a + a*Csc[e + f*x]]])/f

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\csc(e+fx)}\,\sqrt{a+a\csc(e+fx)}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \frac{a\cot(e+fx)}{\sqrt{a+a\csc(e+fx)}}\right)}{f}\\ &= -\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\cot(e+fx)}{\sqrt{a+a\csc(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(37) = 74.
time = 0.44, size = 108, normalized size = 2.92

$$\frac{2\cot(e+fx)\sqrt{a(1+\csc(e+fx))}\left(\log(1+\csc(e+fx))-\log\left(\sqrt{\csc(e+fx)}+\csc^{\frac{3}{2}}(e+fx)+\sqrt{\cot^2(e+fx)}\sqrt{1+\csc(e+fx)}\right)\right)}{f\sqrt{\cot^2(e+fx)}\sqrt{1+\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[e + f*x]]*Sqrt[a + a*Csc[e + f*x]],x]
```

```
[Out] (2*Cot[e + f*x]*Sqrt[a*(1 + Csc[e + f*x])]*(Log[1 + Csc[e + f*x]] - Log[Sqr
t[Csc[e + f*x]] + Csc[e + f*x]^(3/2) + Sqrt[Cot[e + f*x]^2]*Sqrt[1 + Csc[e
+ f*x]]]))/(f*Sqrt[Cot[e + f*x]^2]*Sqrt[1 + Csc[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(31) = 62.
time = 1.34, size = 114, normalized size = 3.08

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\frac{1}{\sin(fx+e)}}\,(\cos(fx+e)-1)\sqrt{\frac{a(\sin(fx+e)+1)}{\sin(fx+e)}}\left(\operatorname{arcsinh}\left(\frac{\cos(fx+e)-1}{\sin(fx+e)}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{1}{1+\cos(fx+e)}}}\right)\right)}{f(-1+\cos(fx+e)-\sin(fx+e))\sqrt{\frac{1}{1+\cos(fx+e)}}}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^(1/2)*(a+a*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*2^(1/2)*(1/sin(f*x+e))^(1/2)*(cos(f*x+e)-1)*(a*(sin(f*x+e)+1)/sin(f*x+
e))^(1/2)*(arcsinh((cos(f*x+e)-1)/sin(f*x+e))+arctanh(1/2*2^(1/2)/(1/(1+cos
(f*x+e)))^(1/2)))/(-1+cos(f*x+e)-sin(f*x+e))/(1/(1+cos(f*x+e)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^(1/2)*(a+a*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*csc(f*x + e) + a)*sqrt(csc(f*x + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(33) = 66.

time = 4.12, size = 309, normalized size = 8.35

$$\left[\frac{\sqrt{a}\,\log\left(\frac{a\cos(fx+e)^3-7\,a\cos(fx+e)^2-9\,a\cos(fx+e)+\left(a\cos(fx+e)^2+8\,a\cos(fx+e)-a\right)\sin(fx+e)+\frac{4\left(\cos(fx+e)^3+3\,\cos(fx+e)^2-\left(\cos(fx+e)^2-2\,\cos(fx+e)-3\right)\sin(fx+e)-\cos(fx+e)-3\right)\sqrt{a}\,\sqrt{\frac{a\sin(fx+e)+a}{\sin(fx+e)}}}{\sqrt{\sin(fx+e)}}-a}{\cos(fx+e)^3+\cos(fx+e)^2+\left(\cos(fx+e)^2-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)}{2\,f}, \frac{\sqrt{-a}\,\arctan\left(\frac{\left(\cos(fx+e)^2+2\,\sin(fx+e)-1\right)\sqrt{-a}\,\sqrt{\frac{a\sin(fx+e)+a}{\sin(fx+e)}}}{2\,a\cos(fx+e)\sqrt{\sin(fx+e)}}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^(1/2)*(a+a*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 9*a*cos(f*x + e)
+ (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) + 4*(cos(f*x + e)^
3 + 3*cos(f*x + e)^2 - (cos(f*x + e)^2 - 2*cos(f*x + e) - 3)*sin(f*x + e) -
 cos(f*x + e) - 3)*sqrt(a)*sqrt((a*sin(f*x + e) + a)/sin(f*x + e))/sqrt(sin
(f*x + e)) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin
(f*x + e) - cos(f*x + e) - 1))/f, sqrt(-a)*arctan(1/2*(cos(f*x + e)^2 + 2*s
in(f*x + e) - 1)*sqrt(-a)*sqrt((a*sin(f*x + e) + a)/sin(f*x + e))/(a*cos(f*
x + e)*sqrt(sin(f*x + e))))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\csc\left(e+fx\right)+1\right)}\,\sqrt{\csc\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**(1/2)*(a+a*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(csc(e + f*x) + 1))*sqrt(csc(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(31) = 62.
time = 0.81, size = 111, normalized size = 3.00

$$\frac{a\left(\frac{2\,\arctan\left(-\frac{a^{\frac{3}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^3}}{\sqrt{-a}\,a}\right)}{\sqrt{-a}}-\frac{\log\left(\left|-a^{\frac{3}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+\sqrt{a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^3}\right|\right)}{\sqrt{a}}\right)\mathrm{sgn}(\sin\left(fx+e\right))}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^(1/2)*(a+a*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] a*(2*arctan(-(a^(3/2)*tan(1/2*f*x + 1/2*e) - sqrt(a^3*tan(1/2*f*x + 1/2*e)^
2 + a^3))/(sqrt(-a)*a))/sqrt(-a) - log(abs(-a^(3/2)*tan(1/2*f*x + 1/2*e) +
sqrt(a^3*tan(1/2*f*x + 1/2*e)^2 + a^3)))/sqrt(a))*sgn(sin(f*x + e))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a+\frac{a}{\sin\left(e+f\,x\right)}}\,\sqrt{\frac{1}{\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(e + f*x))^(1/2)*(1/sin(e + f*x))^(1/2),x)
```

```
[Out] int((a + a/sin(e + f*x))^(1/2)*(1/sin(e + f*x))^(1/2), x)
```

3.20 $\int \sqrt{-\csc(e+fx)}\,\sqrt{a-a\csc(e+fx)}\,dx$

Optimal. Leaf size=38

$$-\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\cot(e+fx)}{\sqrt{a-a\csc(e+fx)}}\right)}{f}$$

```
[Out] -2*arcsinh(cot(f*x+e)*a^(1/2)/(a-a*csc(f*x+e))^(1/2))*a^(1/2)/f
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3886, 221}

$$-\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\cot(e+fx)}{\sqrt{a-a\csc(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Csc[e + f*x]]*Sqrt[a - a*Csc[e + f*x]],x]
[Out] (-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Cot[e + f*x])/Sqrt[a - a*Csc[e + f*x]]])/f
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-\csc(e+fx)}\,\sqrt{a-a\csc(e+fx)}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\cot(e+fx)}{\sqrt{a-a\csc(e+fx)}}\right)}{f}\\ &= -\frac{2\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\cot(e+fx)}{\sqrt{a-a\csc(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 101 vs. $2(38) = 76$.
time = 1.35, size = 101, normalized size = 2.66

$$\frac{2\left(\sinh^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+\tanh^{-1}\left(\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\right)\right)\sqrt{-\csc(e+fx)}\,\sqrt{a-a\csc(e+fx)}\,\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\,\left(-1+\tan\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Csc[e + f*x]]*Sqrt[a - a*Csc[e + f*x]],x]
```

```
[Out] (2*(ArcSinh[Tan[(e + f*x)/2]] + ArcTanh[Sqrt[Sec[(e + f*x)/2]^2]])*Sqrt[-Cs
c[e + f*x]]*Sqrt[a - a*Csc[e + f*x]]*Tan[(e + f*x)/2])/(f*Sqrt[Sec[(e + f*x
)/2]^2]*(-1 + Tan[(e + f*x)/2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(32) = 64$.
time = 1.26, size = 117, normalized size = 3.08

method	result
default	$\frac{2\sqrt{-\frac{1}{\sin(fx+e)}}\,(\cos(fx+e)-1)\sqrt{\frac{a(\sin(fx+e)-1)}{\sin(fx+e)}}\left(\arctan\left(\frac{\sin(fx+e)\sqrt{-\frac{2}{1+\cos(fx+e)}}}{2}\right)-\arctan\left(\frac{1}{\sqrt{-\frac{2}{1+\cos(fx+e)}}}\right.\right.}{f(-1+\cos(fx+e)+\sin(fx+e))\sqrt{-\frac{2}{1+\cos(fx+e)}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(f*x+e))^(1/2)*(a-a*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(-1/sin(f*x+e))^(1/2)*(cos(f*x+e)-1)*(a*(sin(f*x+e)-1)/sin(f*x+e))^(1/2
)*(arctan(1/2*sin(f*x+e)*(-2/(1+cos(f*x+e)))^(1/2))-arctan(1/(-2/(1+cos(f*x
+e)))^(1/2)))/(-1+cos(f*x+e)+sin(f*x+e))/(-2/(1+cos(f*x+e)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(f*x+e))^(1/2)*(a-a*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*csc(f*x + e) + a)*sqrt(-csc(f*x + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(34) = 68$.
time = 3.85, size = 322, normalized size = 8.47

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7a\cos(fx+e)^2-4\left(\cos(fx+e)^3+3\cos(fx+e)^2+\left(\cos(fx+e)^2-2\cos(fx+e)-3\right)\sin(fx+e)-\cos(fx+e)-3\right)\sqrt{a}\sqrt{\frac{a\sin(fx+e)-a}{\sin(fx+e)}}\sqrt{-\frac{1}{\sin(fx+e)}}-9a\cos(fx+e)-\left(a\cos(fx+e)^2+8a\cos(fx+e)-a\right)\sin(fx+e)-a}{\cos(fx+e)^3+\cos(fx+e)^2-\left(\cos(fx+e)^2-1\right)\sin(fx+e)-\cos(fx+e)-1}\right)}{2f}, \frac{\sqrt{-a}\arctan\left(-\frac{\left(\cos(fx+e)^2-2\sin(fx+e)-1\right)\sqrt{-a}\sqrt{\frac{a\sin(fx+e)-a}{\sin(fx+e)}}\sqrt{-\frac{1}{\sin(fx+e)}}}{2a\cos(fx+e)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(f*x+e))^(1/2)*(a-a*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^3
 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 3)*sin(f*x + e) -
cos(f*x + e) - 3)*sqrt(a)*sqrt((a*sin(f*x + e) - a)/sin(f*x + e))*sqrt(-1/s
in(f*x + e)) - 9*a*cos(f*x + e) - (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)
*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 - (cos(f*x + e)^2 - 1)*
sin(f*x + e) - cos(f*x + e) - 1))/f, sqrt(-a)*arctan(-1/2*(cos(f*x + e)^2 -
 2*sin(f*x + e) - 1)*sqrt(-a)*sqrt((a*sin(f*x + e) - a)/sin(f*x + e))*sqrt(
-1/sin(f*x + e))/(a*cos(f*x + e)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\csc(e+fx)}\,\sqrt{-a\left(\csc(e+fx)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(f*x+e))**(1/2)*(a-a*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-csc(e + f*x))*sqrt(-a*(csc(e + f*x) - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(32) = 64$.
time = 0.85, size = 101, normalized size = 2.66

$$-\frac{\frac{2a\arctan\left(\frac{a^{\frac{3}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\sqrt{a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a^3}}{\sqrt{-a}\,a}\right)}{\sqrt{-a}}-\sqrt{a}\log\left(\left|a^{\frac{3}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\sqrt{a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a^3}\right|\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(f*x+e))^(1/2)*(a-a*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a*arctan((a^(3/2)*tan(1/2*f*x + 1/2*e) + sqrt(a^3*tan(1/2*f*x + 1/2*e)^
2 + a^3))/(sqrt(-a)*a))/sqrt(-a) - sqrt(a)*log(abs(a^(3/2)*tan(1/2*f*x + 1/
2*e) + sqrt(a^3*tan(1/2*f*x + 1/2*e)^2 + a^3))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a-\frac{a}{\sin\left(e+f\,x\right)}}\,\sqrt{-\frac{1}{\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/sin(e + f*x))^(1/2)*(-1/sin(e + f*x))^(1/2),x)
[Out] int((a - a/sin(e + f*x))^(1/2)*(-1/sin(e + f*x))^(1/2), x)
```

3.21 $\int \csc^{\frac{4}{3}}(c+dx)\sqrt{a+a\csc(c+dx)}\,dx$

Optimal. Leaf size=254

$$-\frac{6a\cos(c+dx)\csc^{\frac{4}{3}}(c+dx)}{5d\sqrt{a+a\csc(c+dx)}}-\frac{4\ 3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{1+\sqrt[3]{\csc(c+dx)}-\ldots}{\left(1+\sqrt{3}-\sqrt[3]{\csc\ldots}\right.}}}{5d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)^2}}\,(a-a\ldots}$$

```
[Out] -6/5*a*cos(d*x+c)*csc(d*x+c)^(4/3)/d/(a+a*csc(d*x+c))^(1/2)-4/5*3^(3/4)*a^2
*cot(d*x+c)*(1-csc(d*x+c)^(1/3))*EllipticF((1-csc(d*x+c)^(1/3)-3^(1/2))/(1-
csc(d*x+c)^(1/3)+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((1+csc(
d*x+c)^(1/3)+csc(d*x+c)^(2/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)/d/(a-a
*csc(d*x+c))/(a+a*csc(d*x+c))^(1/2)/((1-csc(d*x+c)^(1/3))/(1-csc(d*x+c)^(1/
3)+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3891, 52, 65, 224}

$$-\frac{4\ 3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}-\frac{6a\cos(c+dx)\csc^{\frac{4}{3}}(c+dx)}{5d\sqrt{a\csc(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^(4/3)*Sqrt[a + a*Csc[c + d*x]],x]
```

```
[Out] (-6*a*Cos[c + d*x]*Csc[c + d*x]^(4/3))/(5*d*Sqrt[a + a*Csc[c + d*x]]) - (4*
3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(1/3))*Sqrt[(1
 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/
3))^2]*EllipticF[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1 + Sqrt[3] - C
sc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(5*d*Sqrt[(1 - Csc[c + d*x]^(1/3))/(1
 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[a + a*Csc[c +
 d*x]])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^{\frac{4}{3}}(c+dx)\sqrt{a+a\csc(c+dx)}\,dx = \frac{(a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{\sqrt[3]{x}}{\sqrt{a-ax}}\,dx, x, \csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}\sqrt{a+a\csc(c+dx)}}$$

$$= -\frac{6a\cos(c+dx)\csc^{\frac{4}{3}}(c+dx)}{5d\sqrt{a+a\csc(c+dx)}} + \frac{(2a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{1}{x^{2/3}\sqrt{a \ldots}}\right.}{5d\sqrt{a-a\csc(c+dx)}\sqrt{\ldots}}$$

$$= -\frac{6a\cos(c+dx)\csc^{\frac{4}{3}}(c+dx)}{5d\sqrt{a+a\csc(c+dx)}} + \frac{(6a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\ldots}}\right.}{5d\sqrt{a-a\csc(c+dx)}\sqrt{\ldots}}$$

$$= -\frac{6a\cos(c+dx)\csc^{\frac{4}{3}}(c+dx)}{5d\sqrt{a+a\csc(c+dx)}} - \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\cot(c+dx)\left(\ldots\right.}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.01, size = 102, normalized size = 0.40

$$-\frac{2\sqrt{a(1+\csc(c+dx))}\left(3\sqrt[3]{\csc(c+dx)}+2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};1-\csc(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{5d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^(4/3)*Sqrt[a + a*Csc[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a*(1 + Csc[c + d*x])]*(3*Csc[c + d*x]^(1/3) + 2*Hypergeometric2F1[
1/2, 2/3, 3/2, 1 - Csc[c + d*x]])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(5
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(\csc^{\frac{4}{3}}(dx+c)\right)\sqrt{a+a\csc(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^(4/3)*(a+a*csc(d*x+c))^(1/2),x)
```

```
[Out] int(csc(d*x+c)^(4/3)*(a+a*csc(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(4/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(4/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(4/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**(4/3)*(a+a*csc(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(4/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\sin\left(c+d\,x\right)}}\,{\left(\frac{1}{\sin\left(c+d\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(4/3),x)
[Out] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(4/3), x)
```

3.22 $\int \sqrt[3]{\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}\,dx$

Optimal. Leaf size=213

$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{1+\sqrt[3]{\csc(c+dx)}+\csc^{\frac{2}{3}}(c+dx)}{\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)^2}}\,F\left(\mathrm{ArcSin}\left(\frac{1-}{1+}\right.\right.}{d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)^2}}\,(a-a\csc(c+dx))\sqrt{a+a\csc(c+dx}}$$

```
[Out] -2*3^(3/4)*a^2*cot(d*x+c)*(1-csc(d*x+c)^(1/3))*EllipticF((1-csc(d*x+c)^(1/3
)-3^(1/2))/(1-csc(d*x+c)^(1/3)+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((1+csc(d*x+c)^(1/3)+csc(d*x+c)^(2/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2
)^(1/2)/d/(a-a*csc(d*x+c))/(a+a*csc(d*x+c))^(1/2)/((1-csc(d*x+c)^(1/3))/(1-
csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3891, 65, 224}

$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,F\left(\mathrm{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^(1/3)*Sqrt[a + a*Csc[c + d*x]],x]
```

```
[Out] (-2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(1/3))*Sqr
t[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc[c + d*x]
^(1/3))^2]*EllipticF[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1 + Sqrt[3]
 - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(d*Sqrt[(1 - Csc[c + d*x]^(1/3))/
(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[a + a*Csc[c
 + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt[3]{\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}\,dx = \frac{(a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{1}{x^{2/3}\sqrt{a-ax}}\,dx, x, \csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}}$$

$$= \frac{(3a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^3}}\,dx, x, \sqrt[3]{\csc(c+dx)}\right)}{d\sqrt{a-a\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}}$$

$$= -\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{1+ \cdots}{(\cdots}}\cdots}{d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}\cdots}{\left(1+\sqrt{3}-\sqrt[3]{\csc(c+\cdots}\right.\cdots}}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.71, size = 46, normalized size = 0.22

$$-\frac{2a\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};1-\csc(c+dx)\right)}{d\sqrt{a(1+\csc(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^(1/3)*Sqrt[a + a*Csc[c + d*x]],x]
```

```
[Out] (-2*a*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 3/2, 1 - Csc[c + d*x]])/(d*S
qrt[a*(1 + Csc[c + d*x])])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\csc^{\frac{1}{3}}\left(dx+c\right)\right)\sqrt{a+a\csc\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^(1/3)*(a+a*csc(d*x+c))^(1/2),x)
[Out] int(csc(d*x+c)^(1/3)*(a+a*csc(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(1/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(1/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\csc\left(c+dx\right)+1\right)}\,\sqrt[3]{\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**(1/3)*(a+a*csc(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(csc(c + d*x) + 1))*csc(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(1/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\sin\left(c+d\,x\right)}}\,{\left(\frac{1}{\sin\left(c+d\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(1/3),x)
```

```
[Out] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(1/3), x)
```

3.23 $\int \frac{\sqrt{a+a\csc(c+dx)}}{\csc^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=254

$$-\frac{3a\cos(c+dx)\sqrt[3]{\csc(c+dx)}}{2d\sqrt{a+a\csc(c+dx)}}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{1+\sqrt[3]{\csc(c+dx)}+\cdots}{\left(1+\sqrt{3}-\sqrt[3]{\csc\cdots}\right.}}}{2d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)^2}}\,(a-a\cdots}$$

```
[Out] -3/2*a*cos(d*x+c)*csc(d*x+c)^(1/3)/d/(a+a*csc(d*x+c))^(1/2)-1/2*3^(3/4)*a^2
*cot(d*x+c)*(1-csc(d*x+c)^(1/3))*EllipticF((1-csc(d*x+c)^(1/3)-3^(1/2))/(1-
csc(d*x+c)^(1/3)+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((1+csc(
d*x+c)^(1/3)+csc(d*x+c)^(2/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)/d/(a-a
*csc(d*x+c))/(a+a*csc(d*x+c))^(1/2)/((1-csc(d*x+c)^(1/3))/(1-csc(d*x+c)^(1/
3)+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3891, 53, 65, 224}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}-\frac{3a\cos(c+dx)\sqrt[3]{\csc(c+dx)}}{2d\sqrt{a\csc(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(2/3),x]
```

```
[Out] (-3*a*Cos[c + d*x]*Csc[c + d*x]^(1/3))/(2*d*Sqrt[a + a*Csc[c + d*x]]) - (3^
(3/4)*Sqrt[2 + Sqrt[3]]*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(1/3))*Sqrt[(1 +
 Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3)
)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1 + Sqrt[3] - Csc
[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(2*d*Sqrt[(1 - Csc[c + d*x]^(1/3))/(1 +
 Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[a + a*Csc[c + d
*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\csc(c+dx)}}{\csc^{\frac{2}{3}}(c+dx)}\,dx = \frac{(a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{1}{x^{5/3}\sqrt{a-ax}}\,dx, x, \csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}}$$

$$= -\frac{3a\cos(c+dx)\sqrt[3]{\csc(c+dx)}}{2d\sqrt{a+a\csc(c+dx)}} + \frac{(a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{1}{x^{2/3}\sqrt{a-ax}}\,dx, x\right.}{4d\sqrt{a-a\csc(c+dx)}\,\sqrt{a+a\csc(c}$$

$$= -\frac{3a\cos(c+dx)\sqrt[3]{\csc(c+dx)}}{2d\sqrt{a+a\csc(c+dx)}} + \frac{(3a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^3}}\,dx, x,\right.}{4d\sqrt{a-a\csc(c+dx)}\,\sqrt{a+a\csc(}$$

$$= -\frac{3a\cos(c+dx)\sqrt[3]{\csc(c+dx)}}{2d\sqrt{a+a\csc(c+dx)}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c-\right.}{2d\sqrt{\frac{1}{(1+\sqrt{}}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.95, size = 110, normalized size = 0.43

$$-\frac{\sqrt{a(1+\csc(c+dx))}\left(3+\csc^{\frac{2}{3}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};1-\csc(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d\csc^{\frac{2}{3}}(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(2/3),x]
```

```
[Out] -1/2*(Sqrt[a*(1 + Csc[c + d*x])]*(3 + Csc[c + d*x]^(2/3)*Hypergeometric2F1[
1/2, 2/3, 3/2, 1 - Csc[c + d*x]])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(d
*Csc[c + d*x]^(2/3)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\csc(dx+c)}}{\csc(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(2/3),x)
```

```
[Out] int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\csc\left(c+dx\right)+1\right)}}{\csc^{\frac{2}{3}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))**(1/2)/csc(d*x+c)**(2/3),x)
[Out] Integral(sqrt(a*(csc(c + d*x) + 1))/csc(c + d*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{a}{\sin\left(c+d\,x\right)}}}{{\left(\frac{1}{\sin\left(c+d\,x\right)}\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(2/3),x)
[Out] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(2/3), x)
```

3.24 $\int \csc^{\frac{5}{3}}(c+dx)\sqrt{a+a\csc(c+dx)}\,dx$

Optimal. Leaf size=514

$$\frac{24a\cot(c+dx)}{7d\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)\sqrt{a+a\csc(c+dx)}}-\frac{6a\cos(c+dx)\csc^{\frac{5}{3}}(c+dx)}{7d\sqrt{a+a\csc(c+dx)}}-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2\,\mathrm{co}}{}$$

```
[Out] -6/7*a*cos(d*x+c)*csc(d*x+c)^(5/3)/d/(a+a*csc(d*x+c))^(1/2)+24/7*a*cot(d*x+
c)/d/(1-csc(d*x+c)^(1/3)+3^(1/2))/(a+a*csc(d*x+c))^(1/2)+8/7*3^(3/4)*a^2*co
t(d*x+c)*(1-csc(d*x+c)^(1/3))*EllipticF((1-csc(d*x+c)^(1/3)-3^(1/2))/(1-csc
(d*x+c)^(1/3)+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((1+csc(d*x+c)^(1/3)+csc(d*x+
c)^(2/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)/d/(a-a*csc(d*x+c))/(a+a*csc
(d*x+c))^(1/2)/((1-csc(d*x+c)^(1/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)-
12/7*3^(1/4)*a^2*cot(d*x+c)*(1-csc(d*x+c)^(1/3))*EllipticE((1-csc(d*x+c)^(1
/3)-3^(1/2))/(1-csc(d*x+c)^(1/3)+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((1+csc(d*x+c)^(1/3)+csc(d*x+c)^(2/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))
^2)^(1/2)/d/(a-a*csc(d*x+c))/(a+a*csc(d*x+c))^(1/2)/((1-csc(d*x+c)^(1/3))/(
1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3891, 52, 65, 309, 224, 1891}

$$\frac{8\sqrt{2}\,3^{3/4}a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,F\left(\mathrm{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{7d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,E\left(\mathrm{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{7d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}-\frac{6a\cos(c+dx)\csc^{\frac{5}{3}}(c+dx)}{7d\sqrt{a\csc(c+dx)+a}}+\frac{24a\cot(c+dx)}{7d\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)\sqrt{a\csc(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^(5/3)*Sqrt[a + a*Csc[c + d*x]],x]
```

```
[Out] (24*a*Cot[c + d*x])/(7*d*(1 + Sqrt[3] - Csc[c + d*x]^(1/3))*Sqrt[a + a*Csc[
c + d*x]]) - (6*a*Cos[c + d*x]*Csc[c + d*x]^(5/3))/(7*d*Sqrt[a + a*Csc[c +
d*x]]) - (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(
1/3))*Sqrt[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc
[c + d*x]^(1/3))^2]*EllipticE[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1
+ Sqrt[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(7*d*Sqrt[(1 - Csc[c + d
*x]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[
a + a*Csc[c + d*x]]) + (8*Sqrt[2]*3^(3/4)*a^2*Cot[c + d*x]*(1 - Csc[c + d*x
]^(1/3))*Sqrt[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] -
Csc[c + d*x]^(1/3))^2]*EllipticF[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/
```

```
(1 + Sqrt[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(7*d*Sqrt[(1 - Csc[c
+ d*x]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sq
rt[a + a*Csc[c + d*x]])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^{\frac{5}{3}}(c+dx)\sqrt{a+a\csc(c+dx)}\,dx = \frac{\left(a^2\cot(c+dx)\right)\text{Subst}\left(\int \frac{x^{2/3}}{\sqrt{a-ax}}\,dx, x, \csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}\sqrt{a+a\csc(c+dx)}}$$

$$= -\frac{6a\cos(c+dx)\csc^{\frac{5}{3}}(c+dx)}{7d\sqrt{a+a\csc(c+dx)}} + \frac{\left(4a^2\cot(c+dx)\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{x}\,\sqrt{}}\right.}{7d\sqrt{a-a\csc(c+dx)}}$$

$$= -\frac{6a\cos(c+dx)\csc^{\frac{5}{3}}(c+dx)}{7d\sqrt{a+a\csc(c+dx)}} + \frac{\left(12a^2\cot(c+dx)\right)\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{7d\sqrt{a-a\csc(c+dx)}}$$

$$= -\frac{6a\cos(c+dx)\csc^{\frac{5}{3}}(c+dx)}{7d\sqrt{a+a\csc(c+dx)}} - \frac{\left(12a^2\cot(c+dx)\right)\text{Subst}\left(\int \frac{1-}{\sqrt{a}}\right.}{7d\sqrt{a-a\csc(c+dx)}}$$

$$= \frac{24a\cot(c+dx)}{7d\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)\sqrt{a+a\csc(c+dx)}} - \frac{6a\cos(c}{7d\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 13.57, size = 120, normalized size = 0.23

$$-\frac{2\sqrt{a(1+\csc(c+dx))}\left(3(4+\csc(c+dx))-8\sqrt[3]{\csc(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{3}{2};1-\csc(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{7d\sqrt[3]{\csc(c+dx)}\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^(5/3)*Sqrt[a + a*Csc[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a*(1 + Csc[c + d*x])]*(3*(4 + Csc[c + d*x]) - 8*Csc[c + d*x]^(1/3)
*Hypergeometric2F1[1/2, 4/3, 3/2, 1 - Csc[c + d*x]])*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]))/(7*d*Csc[c + d*x]^(1/3)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\csc^{\frac{5}{3}}\left(dx+c\right)\right) \sqrt{a+a\csc\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^(5/3)*(a+a*csc(d*x+c))^(1/2),x)
```

```
[Out] int(csc(d*x+c)^(5/3)*(a+a*csc(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(5/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(5/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(5/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**(5/3)*(a+a*csc(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(5/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\sin\left(c+d\,x\right)}}\,{\left(\frac{1}{\sin\left(c+d\,x\right)}\right)}^{5/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(5/3),x)
[Out] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(5/3), x)
```

3.25 $\int \csc^{\frac{2}{3}}(c+dx)\sqrt{a+a\csc(c+dx)}\,dx$

Optimal. Leaf size=470

$$\frac{6a\cot(c+dx)}{d\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)\sqrt{a+a\csc(c+dx)}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\cdots}{d\sqrt{\frac{1-\sqrt[3]{\csc(c\cdots}}{\left(1+\sqrt{3}-\sqrt[3]{\csc\cdots}\right.}}\cdots}$$

```
[Out] 6*a*cot(d*x+c)/d/(1-csc(d*x+c)^(1/3)+3^(1/2))/(a+a*csc(d*x+c))^(1/2)+2*3^(3
/4)*a^2*cot(d*x+c)*(1-csc(d*x+c)^(1/3))*EllipticF((1-csc(d*x+c)^(1/3)-3^(1/
2))/(1-csc(d*x+c)^(1/3)+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((1+csc(d*x+c)^(1/3
)+csc(d*x+c)^(2/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)/d/(a-a*csc(d*x+c)
)/(a+a*csc(d*x+c))^(1/2)/((1-csc(d*x+c)^(1/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))
^2)^(1/2)-3*3^(1/4)*a^2*cot(d*x+c)*(1-csc(d*x+c)^(1/3))*EllipticE((1-csc(d*
x+c)^(1/3)-3^(1/2))/(1-csc(d*x+c)^(1/3)+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2
)-1/2*2^(1/2))*((1+csc(d*x+c)^(1/3)+csc(d*x+c)^(2/3))/(1-csc(d*x+c)^(1/3)+3
^(1/2))^2)^(1/2)/d/(a-a*csc(d*x+c))/(a+a*csc(d*x+c))^(1/2)/((1-csc(d*x+c)^(
1/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3891, 65, 309, 224, 1891}

$$\frac{2\sqrt{2}\,3^{3/4}a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}+\frac{6a\cot(c+dx)}{d\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)\sqrt{a\csc(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^(2/3)*Sqrt[a + a*Csc[c + d*x]],x]
```

```
[Out] (6*a*Cot[c + d*x])/(d*(1 + Sqrt[3] - Csc[c + d*x]^(1/3))*Sqrt[a + a*Csc[c +
 d*x]]) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(
1/3))*Sqrt[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc
[c + d*x]^(1/3))^2]*EllipticE[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1
+ Sqrt[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(d*Sqrt[(1 - Csc[c + d*x
]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[a
+ a*Csc[c + d*x]]) + (2*Sqrt[2]*3^(3/4)*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^
(1/3))*Sqrt[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Cs
c[c + d*x]^(1/3))^2]*EllipticF[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1
 + Sqrt[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(d*Sqrt[(1 - Csc[c + d*
```

```
x]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[a
 + a*Csc[c + d*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^{\frac{2}{3}}(c+dx)\sqrt{a+a\csc(c+dx)}\,dx = \frac{(a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt{a-ax}}\,dx, x, \csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}\sqrt{a+a\csc(c+dx)}}$$

$$= \frac{(3a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{x}{\sqrt{a-ax^3}}\,dx, x, \sqrt[3]{\csc(c+dx)}\right)}{d\sqrt{a-a\csc(c+dx)}\sqrt{a+a\csc(c+dx)}}$$

$$= -\frac{(3a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{1-\sqrt{3}-x}{\sqrt{a-ax^3}}\,dx, x, \sqrt[3]{\csc(c+dx)}\right)}{d\sqrt{a-a\csc(c+dx)}\sqrt{a+a\csc(c+dx)}} - $$

$$= \frac{6a\cot(c+dx)}{d\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)\sqrt{a+a\csc(c+dx)}} - \frac{3\sqrt[4]{3}\sqrt{2-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.82, size = 109, normalized size = 0.23

$$\frac{2\sqrt{a(1+\csc(c+dx))}\left(-3+2\sqrt[3]{\csc(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{3}{2};1-\csc(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt[3]{\csc(c+dx)}\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^(2/3)*Sqrt[a + a*Csc[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Csc[c + d*x])]*(-3 + 2*Csc[c + d*x]^(1/3)*Hypergeometric2F1[
1/2, 4/3, 3/2, 1 - Csc[c + d*x]])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(d
*Csc[c + d*x]^(1/3)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\csc^{\frac{2}{3}}(dx+c)\right)\sqrt{a+a\csc(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^(2/3)*(a+a*csc(d*x+c))^(1/2),x)
```

```
[Out] int(csc(d*x+c)^(2/3)*(a+a*csc(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(2/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(2/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\csc\left(c+dx\right)+1\right)}\,\csc^{\frac{2}{3}}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**(2/3)*(a+a*csc(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(csc(c + d*x) + 1))*csc(c + d*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^(2/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{a}{\sin\left(c+d\,x\right)}}\,{\left(\frac{1}{\sin\left(c+d\,x\right)}\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(2/3),x)
[Out] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(2/3), x)
```

3.26 $\int \frac{\sqrt{a+a\csc(c+dx)}}{\sqrt[3]{\csc(c+dx)}}\,dx$

Optimal. Leaf size=508

$$-\frac{3a\cot(c+dx)}{d\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)\sqrt{a+a\csc(c+dx)}}-\frac{3a\cos(c+dx)\csc^{\frac{2}{3}}(c+dx)}{d\sqrt{a+a\csc(c+dx)}}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2\cot(}{}$$

```
[Out] -3*a*cos(d*x+c)*csc(d*x+c)^(2/3)/d/(a+a*csc(d*x+c))^(1/2)-3*a*cot(d*x+c)/d/
(1-csc(d*x+c)^(1/3)+3^(1/2))/(a+a*csc(d*x+c))^(1/2)-3^(3/4)*a^2*cot(d*x+c)*
(1-csc(d*x+c)^(1/3))*EllipticF((1-csc(d*x+c)^(1/3)-3^(1/2))/(1-csc(d*x+c)^(
1/3)+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((1+csc(d*x+c)^(1/3)+csc(d*x+c)^(2/3))
/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)/d/(a-a*csc(d*x+c))/(a+a*csc(d*x+c))^
(1/2)/((1-csc(d*x+c)^(1/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)+3/2*3^(1/
4)*a^2*cot(d*x+c)*(1-csc(d*x+c)^(1/3))*EllipticE((1-csc(d*x+c)^(1/3)-3^(1/2
))/(1-csc(d*x+c)^(1/3)+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((
1+csc(d*x+c)^(1/3)+csc(d*x+c)^(2/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)/
d/(a-a*csc(d*x+c))/(a+a*csc(d*x+c))^(1/2)/((1-csc(d*x+c)^(1/3))/(1-csc(d*x+
c)^(1/3)+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3891, 53, 65, 309, 224, 1891}

$$-\frac{\sqrt{2}\,3^{3/4}a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}\,(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}-\frac{3a\cos(c+dx)\csc^{\frac{2}{3}}(c+dx)}{d\sqrt{a\csc(c+dx)+a}}-\frac{3a\cot(c+dx)}{d\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)\sqrt{a\csc(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(1/3),x]
```

```
[Out] (-3*a*Cot[c + d*x])/(d*(1 + Sqrt[3] - Csc[c + d*x]^(1/3))*Sqrt[a + a*Csc[c
+ d*x]]) - (3*a*Cos[c + d*x]*Csc[c + d*x]^(2/3))/(d*Sqrt[a + a*Csc[c + d*x]
]) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(1/3))
*Sqrt[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc[c +
d*x]^(1/3))^2]*EllipticE[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1 + Sqr
t[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(2*d*Sqrt[(1 - Csc[c + d*x]^(
1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[a + a
*Csc[c + d*x]]) - (Sqrt[2]*3^(3/4)*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(1/3)
```

```
)*Sqrt[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc[c +
 d*x]^(1/3))^2]*EllipticF[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1 + Sq
rt[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(d*Sqrt[(1 - Csc[c + d*x]^(1
/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[a + a*
Csc[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\csc(c+dx)}}{\sqrt[3]{\csc(c+dx)}}\,dx = \frac{\left(a^2\cot(c+dx)\right)\text{Subst}\left(\int \frac{1}{x^{4/3}\sqrt{a-ax}}\,dx, x, \csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}}$$

$$= -\frac{3a\cos(c+dx)\csc^{\frac{2}{3}}(c+dx)}{d\sqrt{a+a\csc(c+dx)}} - \frac{\left(a^2\cot(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,\sqrt{a-ax}}\,dx, x, \ldots\right.}{2d\sqrt{a-a\csc(c+dx)}\,\sqrt{a+a\csc(c+\ldots}}$$

$$= -\frac{3a\cos(c+dx)\csc^{\frac{2}{3}}(c+dx)}{d\sqrt{a+a\csc(c+dx)}} - \frac{\left(3a^2\cot(c+dx)\right)\text{Subst}\left(\int \frac{x}{\sqrt{a-ax^3}}\,dx, x, \sqrt[3]{\ldots}\right.}{2d\sqrt{a-a\csc(c+dx)}\,\sqrt{a+a\csc(c\ldots}}$$

$$= -\frac{3a\cos(c+dx)\csc^{\frac{2}{3}}(c+dx)}{d\sqrt{a+a\csc(c+dx)}} + \frac{\left(3a^2\cot(c+dx)\right)\text{Subst}\left(\int \frac{1-\sqrt{3}-x}{\sqrt{a-ax^3}}\,dx, x, \sqrt[3]{\ldots}\right.}{2d\sqrt{a-a\csc(c+dx)}\,\sqrt{a+a\csc(c\ldots}}$$

$$= -\frac{3a\cot(c+dx)}{d\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)\sqrt{a+a\csc(c+dx)}} - \frac{3a\cos(c+dx)\csc^{\frac{2}{3}}(c\ldots}{d\sqrt{a+a\csc(c+d\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.60, size = 46, normalized size = 0.09

$$-\frac{2a\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{3}{2};1-\csc(c+dx)\right)}{d\sqrt{a(1+\csc(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(1/3),x]
```

```
[Out] (-2*a*Cot[c + d*x]*Hypergeometric2F1[1/2, 4/3, 3/2, 1 - Csc[c + d*x]])/(d*S
qrt[a*(1 + Csc[c + d*x])])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\csc\left(dx+c\right)}}{\csc\left(dx+c\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(1/3),x)
```

```
[Out] int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\csc\left(c+dx\right)+1\right)}}{\sqrt[3]{\csc\left(c+dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))**(1/2)/csc(d*x+c)**(1/3),x)
```

```
[Out] Integral(sqrt(a*(csc(c + d*x) + 1))/csc(c + d*x)**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{a}{\sin\left(c+d\,x\right)}}}{{\left(\frac{1}{\sin\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(1/3),x)
[Out] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(1/3), x)
```

3.27 $\int \frac{\sqrt{a+a\csc(c+dx)}}{\csc^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=552

$$-\frac{15a\cot(c+dx)}{8d\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)\sqrt{a+a\csc(c+dx)}}-\frac{3a\cos(c+dx)}{4d\sqrt[3]{\csc(c+dx)}\sqrt{a+a\csc(c+dx)}}-\frac{15a\cos(c-}{8d\sqrt{a-}}$$

[Out] -3/4*a*cos(d*x+c)/d/csc(d*x+c)^(1/3)/(a+a*csc(d*x+c))^(1/2)-15/8*a*cos(d*x+c)*csc(d*x+c)^(2/3)/d/(a+a*csc(d*x+c))^(1/2)-15/8*a*cot(d*x+c)/d/(1-csc(d*x+c)^(1/3)+3^(1/2))/(a+a*csc(d*x+c))^(1/2)-5/8*3^(3/4)*a^2*cot(d*x+c)*(1-csc(d*x+c)^(1/3))*EllipticF((1-csc(d*x+c)^(1/3)-3^(1/2))/(1-csc(d*x+c)^(1/3)+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((1+csc(d*x+c)^(1/3)+csc(d*x+c)^(2/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)/d/(a-a*csc(d*x+c))/(a+a*csc(d*x+c))^(1/2)/((1-csc(d*x+c)^(1/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)+15/16*3^(1/4)*a^2*cot(d*x+c)*(1-csc(d*x+c)^(1/3))*EllipticE((1-csc(d*x+c)^(1/3)-3^(1/2))/(1-csc(d*x+c)^(1/3)+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((1+csc(d*x+c)^(1/3)+csc(d*x+c)^(2/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)/d/(a-a*csc(d*x+c))/(a+a*csc(d*x+c))^(1/2)/((1-csc(d*x+c)^(1/3))/(1-csc(d*x+c)^(1/3)+3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.23, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3891, 53, 65, 309, 224, 1891}

$$-\frac{5\,3^{3/4}a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2\cot(c+dx)\left(1-\sqrt[3]{\csc(c+dx)}\right)\sqrt{\frac{\csc^{\frac{2}{3}}(c+dx)+\sqrt[3]{\csc(c+dx)}+1}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\csc(c+dx)}-\sqrt{3}+1}{-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{16d\sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)^2}}(a-a\csc(c+dx))\sqrt{a\csc(c+dx)+a}}-\frac{15a\cos(c+dx)\csc^{\frac{2}{3}}(c+dx)}{8d\sqrt{a\csc(c+dx)+a}}-\frac{3a\cos(c+dx)}{4d\sqrt[3]{\csc(c+dx)}\sqrt{a\csc(c+dx)+a}}-\frac{15a\cot(c+dx)}{8d\left(-\sqrt[3]{\csc(c+dx)}+\sqrt{3}+1\right)\sqrt{a\csc(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(4/3),x]

[Out] (-15*a*Cot[c + d*x])/(8*d*(1 + Sqrt[3] - Csc[c + d*x]^(1/3))*Sqrt[a + a*Csc[c + d*x]]) - (3*a*Cos[c + d*x])/(4*d*Csc[c + d*x]^(1/3)*Sqrt[a + a*Csc[c + d*x]]) - (15*a*Cos[c + d*x]*Csc[c + d*x]^(2/3))/(8*d*Sqrt[a + a*Csc[c + d*x]]) + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(1/3))*Sqrt[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*EllipticE[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(16*d*Sqrt[(1 - Csc[c + d*

```
x]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[a
 + a*Csc[c + d*x]]) - (5*3^(3/4)*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(1/3))*
Sqrt[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc[c + d
*x]^(1/3))^2]*EllipticF[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1 + Sqrt
[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*d*Sqrt[(1 - Csc[c +
 d*x]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqr
t[a + a*Csc[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
```

```
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\csc(c+dx)}}{\csc^{\frac{4}{3}}(c+dx)}\,dx = \frac{\left(a^2\cot(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{x^{7/3}\sqrt{a-ax}}\,dx, x, \csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}}$$

$$= -\frac{3a\cos(c+dx)}{4d\sqrt[3]{\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}} + \frac{\left(5a^2\cot(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{x^{4/3}\sqrt{a} \cdots}\right.}{8d\sqrt{a-a\csc(c+dx)}\,\sqrt{\cdots}}$$

$$= -\frac{3a\cos(c+dx)}{4d\sqrt[3]{\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}} - \frac{15a\cos(c+dx)\csc^{\frac{2}{3}}(c+dx)}{8d\sqrt{a+a\csc(c+dx)}} - \frac{(5\cdots}{\cdots}$$

$$= -\frac{3a\cos(c+dx)}{4d\sqrt[3]{\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}} - \frac{15a\cos(c+dx)\csc^{\frac{2}{3}}(c+dx)}{8d\sqrt{a+a\csc(c+dx)}} - \frac{(1\cdots}{\cdots}$$

$$= -\frac{3a\cos(c+dx)}{4d\sqrt[3]{\csc(c+dx)}\,\sqrt{a+a\csc(c+dx)}} - \frac{15a\cos(c+dx)\csc^{\frac{2}{3}}(c+dx)}{8d\sqrt{a+a\csc(c+dx)}} + \frac{(1\cdots}{\cdots}$$

$$= -\frac{15a\cot(c+dx)}{8d\left(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)}\right)\sqrt{a+a\csc(c+dx)}} - \frac{3a\cos(c\cdots}{4d\sqrt[3]{\csc(c+dx)}\,\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 8.03, size = 72, normalized size = 0.13

$$-\frac{a\cos(c+dx)\left(3+5\csc^{\frac{4}{3}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{3}{2};1-\csc(c+dx)\right)\right)}{4d\sqrt[3]{\csc(c+dx)}\,\sqrt{a(1+\csc(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(4/3),x]
```

```
[Out] -1/4*(a*Cos[c + d*x]*(3 + 5*Csc[c + d*x]^(4/3)*Hypergeometric2F1[1/2, 4/3,
3/2, 1 - Csc[c + d*x]]))/(d*Csc[c + d*x]^(1/3)*Sqrt[a*(1 + Csc[c + d*x])])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\csc\left(dx+c\right)}}{\csc\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(4/3),x)
```

```
[Out] int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(4/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\csc\left(c+dx\right)+1\right)}}{\csc^{\frac{4}{3}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))**(1/2)/csc(d*x+c)**(4/3),x)
[Out] Integral(sqrt(a*(csc(c + d*x) + 1))/csc(c + d*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(4/3),x, algorithm="giac")
[Out] integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{a}{\sin\left(c+d\,x\right)}}}{{\left(\frac{1}{\sin\left(c+d\,x\right)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(4/3),x)
[Out] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(4/3), x)
```

3.28 $\int \csc^n(c+dx)\sqrt{a+a\csc(c+dx)}\,dx$

Optimal. Leaf size=48

$$-\frac{2a\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\csc(c+dx)\right)}{d\sqrt{a+a\csc(c+dx)}}$$

```
[Out] -2*a*cot(d*x+c)*hypergeom([1/2, 1-n],[3/2],1-csc(d*x+c))/d/(a+a*csc(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3891, 67}

$$-\frac{2a\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\csc(c+dx)\right)}{d\sqrt{a\csc(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^n*Sqrt[a + a*Csc[c + d*x]],x]
```

```
[Out] (-2*a*Cot[c + d*x]*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Csc[c + d*x]])/(d
*Sqrt[a + a*Csc[c + d*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^n(c+dx)\sqrt{a+a\csc(c+dx)}\,dx &= \frac{\left(a^2\cot(c+dx)\right)\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a-ax}}\,dx,x,\csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}\sqrt{a+a\csc(c+dx)}}\\ &= -\frac{2a\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\csc(c+dx)\right)}{d\sqrt{a+a\csc(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.68, size = 48, normalized size = 1.00

$$-\frac{2a\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\csc(c+dx)\right)}{d\sqrt{a(1+\csc(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^n*Sqrt[a + a*Csc[c + d*x]],x]
[Out] (-2*a*Cot[c + d*x]*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Csc[c + d*x]])/(d
*Sqrt[a*(1 + Csc[c + d*x])])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\csc^{n}\left(dx+c\right)\right)\sqrt{a+a\csc\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^n*(a+a*csc(d*x+c))^(1/2),x)
[Out] int(csc(d*x+c)^n*(a+a*csc(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^n*(a+a*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^n*(a+a*csc(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\csc\left(c+dx\right)+1\right)}\csc^{n}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**n*(a+a*csc(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(csc(c + d*x) + 1))*csc(c + d*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^n*(a+a*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+\frac{a}{\sin\left(c+d\,x\right)}}\,{\left(\frac{1}{\sin\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^n,x)
```

```
[Out] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^n, x)
```

3.29 $\int \csc^n(c+dx)\sqrt{a-a\csc(c+dx)}\,dx$

Optimal. Leaf size=69

$$-\frac{2a\cos(c+dx)(-\csc(c+dx))^{-n}\csc^{1+n}(c+dx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}}$$

```
[Out] -2*a*cos(d*x+c)*csc(d*x+c)^(1+n)*hypergeom([1/2, 1-n],[3/2],1+csc(d*x+c))/d
/((-csc(d*x+c))^n)/(a-a*csc(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3891, 69, 67}

$$-\frac{2a\cos(c+dx)(-\csc(c+dx))^{-n}\csc^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\csc(c+dx)+1\right)}{d\sqrt{a-a\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^n*Sqrt[a - a*Csc[c + d*x]],x]
```

```
[Out] (-2*a*Cos[c + d*x]*Csc[c + d*x]^(1 + n)*Hypergeometric2F1[1/2, 1 - n, 3/2,
1 + Csc[c + d*x]])/(d*(-Csc[c + d*x])^n*Sqrt[a - a*Csc[c + d*x]])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rule 3891

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^n(c+dx)\sqrt{a-a\csc(c+dx)}\,dx = \frac{(a^2\cot(c+dx))\,\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a+ax}}\,dx, x, \csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}\sqrt{a+a\csc(c+dx)}}$$
$$= -\frac{(a^2\cos(c+dx)(-\csc(c+dx))^{-n}\csc^{1+n}(c+dx))\,\text{Subst}\left(\int \frac{(-x)^{-}}{\sqrt{a+}}\right.}{d\sqrt{a-a\csc(c+dx)}\sqrt{a+a\csc(c+dx)}}$$
$$= -\frac{2a\cos(c+dx)(-\csc(c+dx))^{-n}\csc^{1+n}(c+dx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1\right.}{d\sqrt{a-a\csc(c+dx)}}$$

Mathematica [**A**]
time = 1.89, size = 73, normalized size = 1.06

$$-\frac{2a\cos(c+dx)\csc^{1+2n}(c+dx)\left(-\csc^2(c+dx)\right)^{-n}{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\csc(c+dx)\right)}{d\sqrt{a-a\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^n*Sqrt[a - a*Csc[c + d*x]],x]
[Out] (-2*a*Cos[c + d*x]*Csc[c + d*x]^(1 + 2*n)*Hypergeometric2F1[1/2, 1 - n, 3/2
, 1 + Csc[c + d*x]])/(d*(-Csc[c + d*x]^2)^n*Sqrt[a - a*Csc[c + d*x]])
```

Maple [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(\csc^n(dx+c)\right)\sqrt{a-a\csc(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^n*(a-a*csc(d*x+c))^(1/2),x)
[Out] int(csc(d*x+c)^n*(a-a*csc(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^n*(a-a*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*csc(d*x + c) + a)*csc(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^n*(a-a*csc(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a*csc(d*x + c) + a)*csc(d*x + c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\csc\left(c+dx\right)-1\right)}\csc^{n}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**n*(a-a*csc(d*x+c))**(1/2),x)
[Out] Integral(sqrt(-a*(csc(c + d*x) - 1))*csc(c + d*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^n*(a-a*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a*csc(d*x + c) + a)*csc(d*x + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{a-\frac{a}{\sin\left(c+d\,x\right)}}\,\left(\frac{1}{\sin\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^n,x)
[Out] int((a - a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^n, x)
```

3.30 $\int \csc^3(e+fx)(a+a\csc(e+fx))^m\,dx$

Optimal. Leaf size=156

$$\frac{\cot(e+fx)(a+a\csc(e+fx))^m}{f\left(2+3m+m^2\right)}-\frac{\cot(e+fx)(a+a\csc(e+fx))^{1+m}}{af(2+m)}-\frac{2^{\frac{1}{2}+m}\left(1+m+m^2\right)\cot(e+fx)(1+$$

```
[Out] cot(f*x+e)*(a+a*csc(f*x+e))^m/f/(m^2+3*m+2)-cot(f*x+e)*(a+a*csc(f*x+e))^(1+
m)/a/f/(2+m)-2^(1/2+m)*(m^2+m+1)*cot(f*x+e)*(1+csc(f*x+e))^(-1/2-m)*(a+a*cs
c(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*csc(f*x+e))/f/(m^2+3*m+2)
```

Rubi [A]
time = 0.14, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3885, 4086, 3913, 3912, 71}

$$-\frac{2^{m+\frac{1}{2}}\left(m^2+m+1\right)\cot(e+fx)(\csc(e+fx)+1)^{-m-\frac{1}{2}}(a\csc(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx))\right)}{f(m+1)(m+2)}+\frac{\cot(e+fx)(a\csc(e+fx)+a)^m}{f\left(m^2+3m+2\right)}-\frac{\cot(e+fx)(a\csc(e+fx)+a)^{m+1}}{af(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + a*Csc[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*(a + a*Csc[e + f*x])^m)/(f*(2 + 3*m + m^2)) - (Cot[e + f*x]*(
a + a*Csc[e + f*x])^(1 + m))/(a*f*(2 + m)) - (2^(1/2 + m)*(1 + m + m^2)*Cot
[e + f*x]*(1 + Csc[e + f*x])^(-1/2 - m)*(a + a*Csc[e + f*x])^m*Hypergeometr
ic2F1[1/2, 1/2 - m, 3/2, (1 - Csc[e + f*x])/2])/(f*(1 + m)*(2 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
```

```
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \csc^3(e+fx)(a+a\csc(e+fx))^m\,dx = -\frac{\cot(e+fx)(a+a\csc(e+fx))^{1+m}}{af(2+m)} + \frac{\int \csc(e+fx)(a(1+m)\ \cdots}{\cdots}$$

$$= \frac{\cot(e+fx)(a+a\csc(e+fx))^m}{f\left(2+3m+m^2\right)} - \frac{\cot(e+fx)(a+a\csc(e+fx\ \cdots}{af(2+m)}$$

$$= \frac{\cot(e+fx)(a+a\csc(e+fx))^m}{f\left(2+3m+m^2\right)} - \frac{\cot(e+fx)(a+a\csc(e+fx\ \cdots}{af(2+m)}$$

$$= \frac{\cot(e+fx)(a+a\csc(e+fx))^m}{f\left(2+3m+m^2\right)} - \frac{\cot(e+fx)(a+a\csc(e+fx\ \cdots}{af(2+m)}$$

$$= \frac{\cot(e+fx)(a+a\csc(e+fx))^m}{f\left(2+3m+m^2\right)} - \frac{\cot(e+fx)(a+a\csc(e+fx\ \cdots}{af(2+m)}$$

Mathematica [A]
time = 4.42, size = 178, normalized size = 1.14

$$-\frac{(a(1+\csc(e+fx)))^m\left((-2+m)m\cot^4\left(\frac{1}{2}(e+fx)\right){}_2F_1\left(-2-m,-2m;-1-m;-\tan\left(\frac{1}{2}(e+fx)\right)\right)+(2+m)\left(m\,{}_2F_1\left(2-m,-2m;3-m;-\tan\left(\frac{1}{2}(e+fx)\right)\right)+2(-2+m)\cot^2\left(\frac{1}{2}(e+fx)\right){}_2F_1\left(-2m,-m;1-m;-\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)^{-2m}}{4f(-2+m)m(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + a*Csc[e + f*x])^m,x]
```

```
[Out] -1/4*((a*(1 + Csc[e + f*x]))^m*((-2 + m)*m*Cot[(e + f*x)/2]^4*Hypergeometri
c2F1[-2 - m, -2*m, -1 - m, -Tan[(e + f*x)/2]] + (2 + m)*(m*Hypergeometric2F
1[2 - m, -2*m, 3 - m, -Tan[(e + f*x)/2]] + 2*(-2 + m)*Cot[(e + f*x)/2]^2*Hy
pergeometric2F1[-2*m, -m, 1 - m, -Tan[(e + f*x)/2]]))*Tan[(e + f*x)/2]^2)/(
f*(-2 + m)*m*(2 + m)*(1 + Tan[(e + f*x)/2])^(2*m))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\csc^{3}\left(f x +e \right)\right) \left(a +a \csc \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+a*csc(f*x+e))^m,x)
```

```
[Out] int(csc(f*x+e)^3*(a+a*csc(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+a*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+a*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\csc{\left(e + f x \right)} + 1\right)\right)^{m} \csc^{3}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+a*csc(f*x+e))**m,x)
```

```
[Out] Integral((a*(csc(e + f*x) + 1))**m*csc(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+a*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\sin(e+f\,x)}\right)^m}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(e + f*x))^m/sin(e + f*x)^3,x)
```

```
[Out] int((a + a/sin(e + f*x))^m/sin(e + f*x)^3, x)
```

3.31 $\int \csc^2(e+fx)(a+a\csc(e+fx))^m\,dx$

Optimal. Leaf size=109

$$-\frac{\cot(e+fx)(a+a\csc(e+fx))^m}{f(1+m)}-\frac{2^{\frac{1}{2}+m}m\cot(e+fx)(1+\csc(e+fx))^{-\frac{1}{2}-m}(a+a\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\right.}{f(1+m)}$$

```
[Out] -cot(f*x+e)*(a+a*csc(f*x+e))^m/f/(1+m)-2^(1/2+m)*m*cot(f*x+e)*(1+csc(f*x+e)
)^(-1/2-m)*(a+a*csc(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*csc(f*x+
e))/f/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3883, 3913, 3912, 71}

$$-\frac{2^{m+\frac{1}{2}}m\cot(e+fx)(\csc(e+fx)+1)^{-m-\frac{1}{2}}(a\csc(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx))\right)}{f(m+1)}-\frac{\cot(e+fx)(a\csc(e+fx)+a)^m}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + a*Csc[e + f*x])^m,x]
```

```
[Out] -((Cot[e + f*x]*(a + a*Csc[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*m*Cot[e
 + f*x]*(1 + Csc[e + f*x])^(-1/2 - m)*(a + a*Csc[e + f*x])^m*Hypergeometric
2F1[1/2, 1/2 - m, 3/2, (1 - Csc[e + f*x])/2])/(f*(1 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \csc^2(e+fx)(a+a\csc(e+fx))^m\,dx = -\frac{\cot(e+fx)(a+a\csc(e+fx))^m}{f(1+m)} + \frac{m\int \csc(e+fx)(a+a\csc(\ldots}{1+m}$$

$$= -\frac{\cot(e+fx)(a+a\csc(e+fx))^m}{f(1+m)} + \frac{(m(1+\csc(e+fx))^{-m}(a \ldots}{}$$

$$= -\frac{\cot(e+fx)(a+a\csc(e+fx))^m}{f(1+m)} + \frac{\left(m\cot(e+fx)(1+\csc(e \ldots\right.}{}$$

$$= -\frac{\cot(e+fx)(a+a\csc(e+fx))^m}{f(1+m)} - \frac{2^{\frac{1}{2}+m} m\cot(e+fx)(1+\csc \ldots}{}$$

Mathematica [A]
time = 1.23, size = 126, normalized size = 1.16

$$-\frac{(a(1+\csc(e+fx)))^m\left((-1+m)\cot^2\left(\frac{1}{2}(e+fx)\right)\,{}_2F_1\left(-1-m,-2m;-m;-\tan\left(\frac{1}{2}(e+fx)\right)\right)+(1+m)\,{}_2F_1\left(1-m,-2m;2-m;-\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)^{-2m}}{2f(-1+m)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + a*Csc[e + f*x])^m,x]
```

```
[Out] -1/2*((a*(1 + Csc[e + f*x]))^m*((-1 + m)*Cot[(e + f*x)/2]^2*Hypergeometric2
F1[-1 - m, -2*m, -m, -Tan[(e + f*x)/2]] + (1 + m)*Hypergeometric2F1[1 - m,
-2*m, 2 - m, -Tan[(e + f*x)/2]])*Tan[(e + f*x)/2])/(f*(-1 + m)*(1 + m)*(1 +
 Tan[(e + f*x)/2])^(2*m))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\csc^2\left(fx+e\right)\right)\left(a+a\csc\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+a*csc(f*x+e))^m,x)
[Out] int(csc(f*x+e)^2*(a+a*csc(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+a*csc(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+a*csc(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\csc(e+fx)+1))^m \csc^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+a*csc(f*x+e))**m,x)
[Out] Integral((a*(csc(e + f*x) + 1))**m*csc(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+a*csc(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\sin(e+f\,x)}\right)^m}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(e + f*x))^m/sin(e + f*x)^2,x)
[Out] int((a + a/sin(e + f*x))^m/sin(e + f*x)^2, x)
```

3.32 $\int \csc(e+fx)(a+a\csc(e+fx))^m\,dx$

Optimal. Leaf size=74

$$-\frac{2^{\frac{1}{2}+m}\cot(e+fx)(1+\csc(e+fx))^{-\frac{1}{2}-m}(a+a\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx))\right)}{f}$$

```
[Out] -2^(1/2+m)*cot(f*x+e)*(1+csc(f*x+e))^(-1/2-m)*(a+a*csc(f*x+e))^m*hypergeom(
[1/2, 1/2-m],[3/2],1/2-1/2*csc(f*x+e))/f
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3913, 3912, 71}

$$-\frac{2^{m+\frac{1}{2}}\cot(e+fx)(\csc(e+fx)+1)^{-m-\frac{1}{2}}(a\csc(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + a*Csc[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*Cot[e + f*x]*(1 + Csc[e + f*x])^(-1/2 - m)*(a + a*Csc[e + f*
x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Csc[e + f*x])/2])/f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int \csc(e+fx)(a+a\csc(e+fx))^m\,dx = \left((1+\csc(e+fx))^{-m}(a+a\csc(e+fx))^m\right)\int \csc(e+fx)(1+\csc$$

$$= \frac{\left(\cot(e+fx)(1+\csc(e+fx))^{-\frac{1}{2}-m}(a+a\csc(e+fx))^m\right)\text{Subst}\Big(}{f\sqrt{1-\csc(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}\cot(e+fx)(1+\csc(e+fx))^{-\frac{1}{2}-m}(a+a\csc(e+fx))^m\,{}_2F}{f}$$

Mathematica [**A**]
time = 0.23, size = 60, normalized size = 0.81

$$-\frac{(a(1+\csc(e+fx)))^m\,{}_2F_1\left(-2m,-m;1-m;-\tan\left(\frac{1}{2}(e+fx)\right)\right)\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)^{-2m}}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + a*Csc[e + f*x])^m,x]
[Out] -(((a*(1 + Csc[e + f*x]))^m*Hypergeometric2F1[-2*m, -m, 1 - m, -Tan[(e + f*
x)/2]])/(f*m*(1 + Tan[(e + f*x)/2])^(2*m)))
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \csc(fx+e)\,(a+a\csc(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+a*csc(f*x+e))^m,x)
[Out] int(csc(f*x+e)*(a+a*csc(f*x+e))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+a*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*csc(f*x + e) + a)^m*csc(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+a*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*csc(f*x + e) + a)^m*csc(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\csc(e+fx)+1))^m \csc(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+a*csc(f*x+e))**m,x)
```

```
[Out] Integral((a*(csc(e + f*x) + 1))**m*csc(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+a*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*csc(f*x + e) + a)^m*csc(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\sin(e+f\,x)}\right)^m}{\sin\left(e+f\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(e + f*x))^m/sin(e + f*x),x)
```

```
[Out] int((a + a/sin(e + f*x))^m/sin(e + f*x), x)
```

3.33 $\int (a + a\csc(e + fx))^m \, dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{2}\, F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\csc(e+fx)),1+\csc(e+fx)\right)\cot(e+fx)(a+a\csc(e+fx))^m}{f(1+2m)\sqrt{1-\csc(e+fx)}}$$

```
[Out] -AppellF1(1/2+m,1,1/2,3/2+m,1+csc(f*x+e),1/2+1/2*csc(f*x+e))*cot(f*x+e)*(a+
a*csc(f*x+e))^m*2^(1/2)/f/(1+2*m)/(1-csc(f*x+e))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3864, 3863, 141}

$$-\frac{\sqrt{2}\,\cot(e+fx)(a\csc(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\csc(e+fx)+1),\csc(e+fx)+1\right)}{f(2m+1)\sqrt{1-\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Csc[e + f*x])^m,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Csc[e + f*x])/2, 1 + Csc
[e + f*x]]*Cot[e + f*x]*(a + a*Csc[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[1 - Csc[e
 + f*x]]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
```

```
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\csc(e+fx))^m\,dx = \left((1+\csc(e+fx))^{-m}(a+a\csc(e+fx))^m\right)\int (1+\csc(e+fx))^m\,dx$$

$$= \frac{\left(\cot(e+fx)(1+\csc(e+fx))^{-\frac{1}{2}-m}(a+a\csc(e+fx))^m\right)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}\,x}\right.}{f\sqrt{1-\csc(e+fx)}}$$

$$= -\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\csc(e+fx)),1+\csc(e+fx)\right)\cot(e+fx)(}{f(1+2m)\sqrt{1-\csc(e+fx)}}$$

Mathematica [F]
time = 0.64, size = 0, normalized size = 0.00

$$\int (a+a\csc(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Csc[e + f*x])^m,x]
[Out] Integrate[(a + a*Csc[e + f*x])^m, x]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (a+a\csc(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*csc(f*x+e))^m,x)
[Out] int((a+a*csc(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*csc(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*csc(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc (e+fx)+a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))**m,x)
[Out] Integral((a*csc(e + f*x) + a)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*csc(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\sin (e+f\,x)}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(e + f*x))^m,x)
[Out] int((a + a/sin(e + f*x))^m, x)
```

3.34 $\int (a+a\csc(e+fx))^m \sin(e+fx)\,dx$

Optimal. Leaf size=83

$$\frac{\sqrt{2}\, F_1\left(\frac{1}{2}+m;\frac{1}{2},2;\frac{3}{2}+m;\frac{1}{2}(1+\csc(e+fx)),1+\csc(e+fx)\right)\cot(e+fx)(a+a\csc(e+fx))^m}{f(1+2m)\sqrt{1-\csc(e+fx)}}$$

```
[Out] AppellF1(1/2+m,2,1/2,3/2+m,1+csc(f*x+e),1/2+1/2*csc(f*x+e))*cot(f*x+e)*(a+a
*csc(f*x+e))^m*2^(1/2)/f/(1+2*m)/(1-csc(f*x+e))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3913, 3912, 141}

$$\frac{\sqrt{2}\cot(e+fx)(a\csc(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\csc(e+fx)+1),\csc(e+fx)+1\right)}{f(2m+1)\sqrt{1-\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Csc[e + f*x])^m*Sin[e + f*x],x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Csc[e + f*x])/2, 1 + Csc[e
 + f*x]]*Cot[e + f*x]*(a + a*Csc[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[1 - Csc[e +
 f*x]])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\csc(e+fx))^m \sin(e+fx)\,dx = \left((1+\csc(e+fx))^{-m}(a+a\csc(e+fx))^m\right)\int (1+\csc(e+fx))^m$$

$$= \frac{\left(\cot(e+fx)(1+\csc(e+fx))^{-\frac{1}{2}-m}(a+a\csc(e+fx))^m\right)\mathrm{Subst}\Big(}{f\sqrt{1-\csc(e+fx)}}$$

$$= \frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},2;\frac{3}{2}+m;\frac{1}{2}(1+\csc(e+fx)),1+\csc(e+fx)\right)\mathrm{co}}{f(1+2m)\sqrt{1-\csc(e+fx)}}$$

Mathematica [F]
time = 3.91, size = 0, normalized size = 0.00

$$\int (a+a\csc(e+fx))^m \sin(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Csc[e + f*x])^m*Sin[e + f*x],x]
[Out] Integrate[(a + a*Csc[e + f*x])^m*Sin[e + f*x], x]
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+a\csc\left(fx+e\right)\right)^{m}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*csc(f*x+e))^m*sin(f*x+e),x)
[Out] int((a+a*csc(f*x+e))^m*sin(f*x+e),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((a*csc(f*x + e) + a)^m*sin(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((a*csc(f*x + e) + a)^m*sin(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\csc(e+fx)+1))^m \sin(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))**m*sin(f*x+e),x)
```

```
[Out] Integral((a*(csc(e + f*x) + 1))**m*sin(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((a*csc(f*x + e) + a)^m*sin(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e+f\,x)\,\left(a+\frac{a}{\sin(e+f\,x)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + a/sin(e + f*x))^m,x)
```

```
[Out] int(sin(e + f*x)*(a + a/sin(e + f*x))^m, x)
```

3.35 $\int (a + a\csc(e + fx))^m \sin^2(e + fx)\, dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{2}\, F_1\left(\frac{1}{2}+m;\frac{1}{2},3;\frac{3}{2}+m;\frac{1}{2}(1+\csc(e+fx)),1+\csc(e+fx)\right)\cot(e+fx)(a+a\csc(e+fx))^m}{f(1+2m)\sqrt{1-\csc(e+fx)}}$$

```
[Out] -AppellF1(1/2+m,3,1/2,3/2+m,1+csc(f*x+e),1/2+1/2*csc(f*x+e))*cot(f*x+e)*(a+
a*csc(f*x+e))^m*2^(1/2)/f/(1+2*m)/(1-csc(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3913, 3912, 141}

$$-\frac{\sqrt{2}\cot(e+fx)(a\csc(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},3;m+\frac{3}{2};\frac{1}{2}(\csc(e+fx)+1),\csc(e+fx)+1\right)}{f(2m+1)\sqrt{1-\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Csc[e + f*x])^m*Sin[e + f*x]^2,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2 + m, 1/2, 3, 3/2 + m, (1 + Csc[e + f*x])/2, 1 + Csc
[e + f*x]]*Cot[e + f*x]*(a + a*Csc[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[1 - Csc[e
 + f*x]]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
```

```
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\int (a+a\csc(e+fx))^m \sin^2(e+fx)\,dx = \left((1+\csc(e+fx))^{-m}(a+a\csc(e+fx))^m\right)\int (1+\csc(e+fx))^m \mathrm{s}$$

$$= \frac{\left(\cot(e+fx)(1+\csc(e+fx))^{-\frac{1}{2}-m}(a+a\csc(e+fx))^m\right)\mathrm{Subst}\Big(}{f\sqrt{1-\csc(e+fx)}}$$

$$= -\frac{\sqrt{2}\,F_1\left(\frac{1}{2}+m;\frac{1}{2},3;\frac{3}{2}+m;\frac{1}{2}(1+\csc(e+fx)),1+\csc(e+fx)\right)}{f(1+2m)\sqrt{1-\csc(e+fx)}}$$

Mathematica [F]
time = 6.82, size = 0, normalized size = 0.00

$$\int (a+a\csc(e+fx))^m \sin^2(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Csc[e + f*x])^m*Sin[e + f*x]^2,x]
[Out] Integrate[(a + a*Csc[e + f*x])^m*Sin[e + f*x]^2, x]
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int (a+a\csc(fx+e))^m \left(\sin^2(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*csc(f*x+e))^m*sin(f*x+e)^2,x)
[Out] int((a+a*csc(f*x+e))^m*sin(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((a*csc(f*x + e) + a)^m*sin(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(a*csc(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\csc(e+fx)+1))^m \sin^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))**m*sin(f*x+e)**2,x)
[Out] Integral((a*(csc(e + f*x) + 1))**m*sin(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="giac")
[Out] integrate((a*csc(f*x + e) + a)^m*sin(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e+f\,x)^2\left(a+\frac{a}{\sin(e+f\,x)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + a/sin(e + f*x))^m,x)
[Out] int(sin(e + f*x)^2*(a + a/sin(e + f*x))^m, x)
```

3.36 $\int (a+b\csc(c+dx))^4\,dx$

Optimal. Leaf size=107

$$a^4x-\frac{2ab(2a^2+b^2)\tanh^{-1}(\cos(c+dx))}{d}-\frac{b^2(17a^2+2b^2)\cot(c+dx)}{3d}-\frac{4ab^3\cot(c+dx)\csc(c+dx)}{3d}-\frac{b^2\cot(c$$

```
[Out] a^4*x-2*a*b*(2*a^2+b^2)*arctanh(cos(d*x+c))/d-1/3*b^2*(17*a^2+2*b^2)*cot(d*
x+c)/d-4/3*a*b^3*cot(d*x+c)*csc(d*x+c)/d-1/3*b^2*cot(d*x+c)*(a+b*csc(d*x+c)
)^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3867, 4133, 3855, 3852, 8}

$$a^4x-\frac{b^2(17a^2+2b^2)\cot(c+dx)}{3d}-\frac{2ab(2a^2+b^2)\tanh^{-1}(\cos(c+dx))}{d}-\frac{4ab^3\cot(c+dx)\csc(c+dx)}{3d}-\frac{b^2\cot(c+dx)(a+b\csc(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csc[c + d*x])^4,x]
```

```
[Out] a^4*x - (2*a*b*(2*a^2 + b^2)*ArcTanh[Cos[c + d*x]])/d - (b^2*(17*a^2 + 2*b^
2)*Cot[c + d*x])/(3*d) - (4*a*b^3*Cot[c + d*x]*Csc[c + d*x])/(3*d) - (b^2*C
ot[c + d*x]*(a + b*Csc[c + d*x])^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+b\csc(c+dx))^4\,dx &= -\frac{b^2\cot(c+dx)(a+b\csc(c+dx))^2}{3d}+\frac{1}{3}\int (a+b\csc(c+dx))\left(3a^3+b\big(9a^2+2\right.\\
&= -\frac{4ab^3\cot(c+dx)\csc(c+dx)}{3d}-\frac{b^2\cot(c+dx)(a+b\csc(c+dx))^2}{3d}+\frac{1}{6}\int (6a^4\\
&= a^4x-\frac{4ab^3\cot(c+dx)\csc(c+dx)}{3d}-\frac{b^2\cot(c+dx)(a+b\csc(c+dx))^2}{3d}+\big(2ab\\
&= a^4x-\frac{2ab(2a^2+b^2)\tanh^{-1}(\cos(c+dx))}{d}-\frac{4ab^3\cot(c+dx)\csc(c+dx)}{3d}-\frac{b^2c}{}\\
&= a^4x-\frac{2ab(2a^2+b^2)\tanh^{-1}(\cos(c+dx))}{d}-\frac{b^2(17a^2+2b^2)\cot(c+dx)}{3d}-\frac{4ab^3}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 568 vs. 2(107) = 214.
time = 6.27, size = 568, normalized size = 5.31

$$\frac{a^4(c+dx)(a+b\csc(c+dx))^4\sin^4(c+dx)}{d(b+a\sin(c+dx))^4}+\frac{\left(-9a^2b^2\cos\left(\frac{1}{2}(c+dx)\right)-b^4\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)(a+b\csc(c+dx))^4\sin^4(c+dx)}{3d(b+a\sin(c+dx))^4}-\frac{ab^3\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\csc(c+dx))^4\sin^4(c+dx)}{2d(b+a\sin(c+dx))^4}-\frac{b^4\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\csc(c+dx))^4\sin^4(c+dx)}{24d(b+a\sin(c+dx))^4}-\frac{2(2a^3b+ab^3)(a+b\csc(c+dx))^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin^4(c+dx)}{d(b+a\sin(c+dx))^4}+\frac{2(2a^3b+ab^3)(a+b\csc(c+dx))^4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin^4(c+dx)}{d(b+a\sin(c+dx))^4}+\frac{ab^3(a+b\csc(c+dx))^4\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^4(c+dx)}{2d(b+a\sin(c+dx))^4}+\frac{(a+b\csc(c+dx))^4\sec\left(\frac{1}{2}(c+dx)\right)\left(9a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)+b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin^4(c+dx)}{3d(b+a\sin(c+dx))^4}+\frac{b^4(a+b\csc(c+dx))^4\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^4(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{24d(b+a\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csc[c + d*x])^4,x]
```

```
[Out] (a^4*(c + d*x)*(a + b*Csc[c + d*x])^4*Sin[c + d*x]^4)/(d*(b + a*Sin[c + d*x
])^4) + ((-9*a^2*b^2*Cos[(c + d*x)/2] - b^4*Cos[(c + d*x)/2])*Csc[(c + d*x)
/2]*(a + b*Csc[c + d*x])^4*Sin[c + d*x]^4)/(3*d*(b + a*Sin[c + d*x])^4) - (
a*b^3*Csc[(c + d*x)/2]^2*(a + b*Csc[c + d*x])^4*Sin[c + d*x]^4)/(2*d*(b + a
*Sin[c + d*x])^4) - (b^4*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + b*Csc[c +
 d*x])^4*Sin[c + d*x]^4)/(24*d*(b + a*Sin[c + d*x])^4) - (2*(2*a^3*b + a*b^
3)*(a + b*Csc[c + d*x])^4*Log[Cos[(c + d*x)/2]]*Sin[c + d*x]^4)/(d*(b + a*S
in[c + d*x])^4) + (2*(2*a^3*b + a*b^3)*(a + b*Csc[c + d*x])^4*Log[Sin[(c +
d*x)/2]]*Sin[c + d*x]^4)/(d*(b + a*Sin[c + d*x])^4) + (a*b^3*(a + b*Csc[c +
```

```
d*x])^4*Sec[(c + d*x)/2]^2*Sin[c + d*x]^4)/(2*d*(b + a*Sin[c + d*x])^4) +
((a + b*Csc[c + d*x])^4*Sec[(c + d*x)/2]*(9*a^2*b^2*Sin[(c + d*x)/2] + b^4*
Sin[(c + d*x)/2])*Sin[c + d*x]^4)/(3*d*(b + a*Sin[c + d*x])^4) + (b^4*(a +
b*Csc[c + d*x])^4*Sec[(c + d*x)/2]^2*Sin[c + d*x]^4*Tan[(c + d*x)/2])/(24*d
*(b + a*Sin[c + d*x])^4)
```

Maple [A]
time = 0.17, size = 112, normalized size = 1.05

method	result
derivativedivides	$\frac{a^4(dx+c)+4a^3b\ln(\csc(dx+c)-\cot(dx+c))-6a^2b^2\cot(dx+c)+4a\,b^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b^4}{d}$
default	$\frac{a^4(dx+c)+4a^3b\ln(\csc(dx+c)-\cot(dx+c))-6a^2b^2\cot(dx+c)+4a\,b^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b^4}{d}$
norman	$\frac{a^4x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^4}{24d}+\frac{b^4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{a\,b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{a\,b^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{3b^2\left(8a^2+b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$
risch	$a^4x+\frac{4b^2\left(-9ia^2e^{4i(dx+c)}+3ab\,e^{5i(dx+c)}+18ia^2e^{2i(dx+c)}+3ib^2e^{2i(dx+c)}-9ia^2-ib^2-3ab\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csc(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(d*x+c)+4*a^3*b*ln(csc(d*x+c)-cot(d*x+c))-6*a^2*b^2*cot(d*x+c)+4*a
*b^3*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c)))+b^4*(-2/3-1
/3*csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.27, size = 125, normalized size = 1.17

$$a^4x+\frac{ab^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{d}-\frac{4\,a^3b\log\left(\cot\left(dx+c\right)+\csc\left(dx+c\right)\right)}{d}-\frac{6\,a^2b^2}{d\tan\left(dx+c\right)}-\frac{\left(3\,\tan\left(dx+c\right)^2+1\right)b^4}{3\,d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + a*b^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1)
+ log(cos(d*x + c) - 1))/d - 4*a^3*b*log(cot(d*x + c) + csc(d*x + c))/d - 6
*a^2*b^2/(d*tan(d*x + c)) - 1/3*(3*tan(d*x + c)^2 + 1)*b^4/(d*tan(d*x + c)^
3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(101) = 202.
time = 3.33, size = 217, normalized size = 2.03

$$-\frac{2\left(9\,a^2b^2+b^4\right)\cos\left(dx+c\right)^3-3\left(2\,a^3b+ab^3-\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(2\,a^3b+ab^3-\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\left(6\,a^2b^2+b^4\right)\cos\left(dx+c\right)-3\left(a^4dx\cos\left(dx+c\right)^2-a^4dx+2\,ab^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(9*a^2*b^2 + b^4)*cos(d*x + c)^3 - 3*(2*a^3*b + a*b^3 - (2*a^3*b +
a*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(2*a^3*
b + a*b^3 - (2*a^3*b + a*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 3*(6*a^2*b^2 + b^4)*cos(d*x + c) - 3*(a^4*d*x*cos(d*x + c)^2
 - a^4*d*x + 2*a*b^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\csc(c + dx))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))**4,x)
```

```
[Out] Integral((a + b*csc(c + d*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(101) = 202.
time = 0.42, size = 205, normalized size = 1.92

$$\frac{b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,(dx+c)a^4+72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+48\,(2\,a^3b+ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{176\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+88\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^4}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(b^4*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 24*(d*
x + c)*a^4 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c) + 9*b^4*tan(1/2*d*x + 1/2*c) +
 48*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) - (176*a^3*b*tan(1/2*d
*x + 1/2*c)^3 + 88*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 72*a^2*b^2*tan(1/2*d*x +
1/2*c)^2 + 9*b^4*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^3*tan(1/2*d*x + 1/2*c) + b
^4)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 0.62, size = 314, normalized size = 2.93

$$\frac{b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{b^4\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{3\,b^4\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{3\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{2\,a^4\,\mathrm{atan}\left(\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)}{d}+\frac{2\,a\,b^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,a^3\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{3\,a^2\,b^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}-\frac{a\,b^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d}+\frac{3\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}+\frac{a\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sin(c + d*x))^4,x)
```

```
[Out] (b^4*tan(c/2 + (d*x)/2)^3)/(24*d) - (b^4*cot(c/2 + (d*x)/2)^3)/(24*d) - (3*
b^4*cot(c/2 + (d*x)/2))/(8*d) + (3*b^4*tan(c/2 + (d*x)/2))/(8*d) + (2*a^4*a
```

```
tan((a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) + 4*a^2*b*sin(c/2 +
(d*x)/2))/(2*b^3*cos(c/2 + (d*x)/2) - a^3*sin(c/2 + (d*x)/2) + 4*a^2*b*cos(
c/2 + (d*x)/2))))/d + (2*a*b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (4*a^3*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (3*a^2*b^2*cot
(c/2 + (d*x)/2))/d - (a*b^3*cot(c/2 + (d*x)/2)^2)/(2*d) + (3*a^2*b^2*tan(c/
2 + (d*x)/2))/d + (a*b^3*tan(c/2 + (d*x)/2)^2)/(2*d)
```

3.37 $\int (a+b\csc(c+dx))^3\,dx$

Optimal. Leaf size=73

$$a^3x-\frac{b(6a^2+b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5ab^2\cot(c+dx)}{2d}-\frac{b^2\cot(c+dx)(a+b\csc(c+dx))}{2d}$$

```
[Out] a^3*x-1/2*b*(6*a^2+b^2)*arctanh(cos(d*x+c))/d-5/2*a*b^2*cot(d*x+c)/d-1/2*b^
2*cot(d*x+c)*(a+b*csc(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3867, 3855, 3852, 8}

$$a^3x-\frac{b(6a^2+b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5ab^2\cot(c+dx)}{2d}-\frac{b^2\cot(c+dx)(a+b\csc(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csc[c + d*x])^3,x]
```

```
[Out] a^3*x - (b*(6*a^2 + b^2)*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a*b^2*Cot[c + d*
x])/(2*d) - (b^2*Cot[c + d*x]*(a + b*Csc[c + d*x]))/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\csc(c+dx))^3\,dx &= -\frac{b^2\cot(c+dx)(a+b\csc(c+dx))}{2d}+\frac{1}{2}\int \left(2a^3+b\left(6a^2+b^2\right)\csc(c+dx)+5ab^2\right. \\
&= a^3x-\frac{b^2\cot(c+dx)(a+b\csc(c+dx))}{2d}+\frac{1}{2}\left(5ab^2\right)\int \csc^2(c+dx)\,dx+\frac{1}{2}\left(b\left(6a^2\right.\right. \\
&= a^3x-\frac{b(6a^2+b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^2\cot(c+dx)(a+b\csc(c+dx))}{2d}-\frac{(5a}{} \\
&= a^3x-\frac{b(6a^2+b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5ab^2\cot(c+dx)}{2d}-\frac{b^2\cot(c+dx)(a+}{2d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 152 vs. 2(73) = 146.
time = 0.69, size = 152, normalized size = 2.08

$$
\frac{8a^3c+8a^3dx-12ab^2\cot\left(\frac{1}{2}(c+dx)\right)-b^3\csc^2\left(\frac{1}{2}(c+dx)\right)-24a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+24a^2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^3\sec^2\left(\frac{1}{2}(c+dx)\right)+12ab^2\tan\left(\frac{1}{2}(c+dx)\right)}{8d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csc[c + d*x])^3,x]
```

```
[Out] (8*a^3*c + 8*a^3*d*x - 12*a*b^2*Cot[(c + d*x)/2] - b^3*Csc[(c + d*x)/2]^2 -
 24*a^2*b*Log[Cos[(c + d*x)/2]] - 4*b^3*Log[Cos[(c + d*x)/2]] + 24*a^2*b*Lo
g[Sin[(c + d*x)/2]] + 4*b^3*Log[Sin[(c + d*x)/2]] + b^3*Sec[(c + d*x)/2]^2
+ 12*a*b^2*Tan[(c + d*x)/2])/(8*d)
```

Maple [A]
time = 0.14, size = 86, normalized size = 1.18

method	result
derivativedivides	$\frac{a^3(dx+c)+3a^2b\ln(\csc(dx+c)-\cot(dx+c))-3a\,b^2\cot(dx+c)+b^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{a^3(dx+c)+3a^2b\ln(\csc(dx+c)-\cot(dx+c))-3a\,b^2\cot(dx+c)+b^3\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
norman	$\frac{a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^3}{8d}+\frac{b^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{3a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{3a\,b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{b(6a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$
risch	$a^3x+\frac{b^2\left(-6ia\,e^{2i(dx+c)}+b\,e^{3i(dx+c)}+6ia+b\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+\frac{3b\ln\left(e^{i(dx+c)}-1\right)a^2}{d}+\frac{b^3\ln\left(e^{i(dx+c)}-1\right)}{2d}-\frac{3b\ln\left(e^{i(dx+}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csc(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(d*x+c)+3*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-3*a*b^2*cot(d*x+c)+b^3*(
-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 95, normalized size = 1.30

$$a^3x+\frac{b^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{4\,d}-\frac{3\,a^2b\log\left(\cot\left(dx+c\right)+\csc\left(dx+c\right)\right)}{d}-\frac{3\,ab^2}{d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/4*b^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1
) + log(cos(d*x + c) - 1))/d - 3*a^2*b*log(cot(d*x + c) + csc(d*x + c))/d -
 3*a*b^2/(d*tan(d*x + c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(67) = 134.
time = 3.48, size = 155, normalized size = 2.12

$$\frac{4\,a^3dx\cos\left(dx+c\right)^2-4\,a^3dx+12\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)+2\,b^3\cos\left(dx+c\right)+\left(6\,a^2b+b^3-\left(6\,a^2b+b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(6\,a^2b+b^3-\left(6\,a^2b+b^3\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{4\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 - 4*a^3*d*x + 12*a*b^2*cos(d*x + c)*sin(d*x +
 c) + 2*b^3*cos(d*x + c) + (6*a^2*b + b^3 - (6*a^2*b + b^3)*cos(d*x + c)^2)
*log(1/2*cos(d*x + c) + 1/2) - (6*a^2*b + b^3 - (6*a^2*b + b^3)*cos(d*x + c
)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\csc\left(c+dx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))**3,x)
```

```
[Out] Integral((a + b*csc(c + d*x))**3, x)
```

Giac [A]
time = 0.42, size = 134, normalized size = 1.84

$$\frac{b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\left(dx+c\right)a^3+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\left(6\,a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(b^3*tan(1/2*d*x + 1/2*c)^2 + 8*(d*x + c)*a^3 + 12*a*b^2*tan(1/2*d*x +
1/2*c) + 4*(6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c))) - (36*a^2*b*tan(1
/2*d*x + 1/2*c)^2 + 6*b^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(1/2*d*x + 1
/2*c) + b^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 0.42, size = 234, normalized size = 3.21

$$\frac{b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{b^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{b^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,d}+\frac{2\,a^3\,\mathrm{atan}\left(\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3+6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{-2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3+6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)}{d}+\frac{3\,a^2\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{3\,a\,b^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{3\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sin(c + d*x))^3,x)
```

```
[Out] (b^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (b^3*cot(c/2 + (d*x)/2)^2)/(8*d) + (b^3*
log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*d) + (2*a^3*atan((2*a^3*cos(
c/2 + (d*x)/2) + b^3*sin(c/2 + (d*x)/2) + 6*a^2*b*sin(c/2 + (d*x)/2))/(b^3*
cos(c/2 + (d*x)/2) - 2*a^3*sin(c/2 + (d*x)/2) + 6*a^2*b*cos(c/2 + (d*x)/2))
))/d + (3*a^2*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (3*a*b^2*co
t(c/2 + (d*x)/2))/(2*d) + (3*a*b^2*tan(c/2 + (d*x)/2))/(2*d)
```

3.38 $\int (a + b\csc(c + dx))^2 \, dx$

Optimal. Leaf size=34

$$a^2x - \frac{2ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{b^2\cot(c+dx)}{d}$$

```
[Out] a^2*x-2*a*b*arctanh(cos(d*x+c))/d-b^2*cot(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3858, 3855, 3852, 8}

$$a^2x - \frac{2ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{b^2\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csc[c + d*x])^2,x]
```

```
[Out] a^2*x - (2*a*b*ArcTanh[Cos[c + d*x]])/d - (b^2*Cot[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\csc(c+dx))^2\,dx &= a^2x+(2ab)\int \csc(c+dx)\,dx+b^2\int \csc^2(c+dx)\,dx\\ &= a^2x-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{b^2\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{d}\\ &= a^2x-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{b^2\cot(c+dx)}{d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.
time = 0.20, size = 76, normalized size = 2.24

$$\frac{-b^2\cot\left(\frac{1}{2}(c+dx)\right)+2a\left(ac+adx-2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+b^2\tan\left(\frac{1}{2}(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csc[c + d*x])^2,x]
```

```
[Out] (-(b^2*Cot[(c + d*x)/2]) + 2*a*(a*c + a*d*x - 2*b*Log[Cos[(c + d*x)/2]] + 2
*b*Log[Sin[(c + d*x)/2]]) + b^2*Tan[(c + d*x)/2])/(2*d)
```

Maple [A]
time = 0.08, size = 46, normalized size = 1.35

method	result	size
derivativedivides	$\frac{a^2(dx+c)+2ab\ln(\csc(dx+c)-\cot(dx+c))-b^2\cot(dx+c)}{d}$	46
default	$\frac{a^2(dx+c)+2ab\ln(\csc(dx+c)-\cot(dx+c))-b^2\cot(dx+c)}{d}$	46
risch	$a^2x-\frac{2ib^2}{d(\mathrm{e}^{2i(dx+c)}-1)}-\frac{2ab\ln(\mathrm{e}^{i(dx+c)}+1)}{d}+\frac{2ab\ln(\mathrm{e}^{i(dx+c)}-1)}{d}$	67
norman	$\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{b^2}{2d}+\frac{b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csc(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(d*x+c)+2*a*b*ln(csc(d*x+c)-cot(d*x+c))-b^2*cot(d*x+c))
```

Maxima [A]
time = 0.29, size = 43, normalized size = 1.26

$$a^2x-\frac{2\,ab\log\left(\cot\left(dx+c\right)+\csc\left(dx+c\right)\right)}{d}-\frac{b^2}{d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 2*a*b*log(cot(d*x + c) + csc(d*x + c))/d - b^2/(d*tan(d*x + c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.
time = 3.40, size = 77, normalized size = 2.26

$$\frac{a^2dx\sin(dx+c) - ab\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + ab\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - b^2\cos(dx+c)}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*sin(d*x + c) - a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + a*b*
log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b^2*cos(d*x + c))/(d*sin(d*x +
c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\csc(c + dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))**2,x)
```

```
[Out] Integral((a + b*csc(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(34) = 68$.
time = 0.42, size = 74, normalized size = 2.18

$$\frac{2(dx+c)a^2 + 4ab\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \frac{4ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^2 + 4*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + b^2*tan(1/2*d
*x + 1/2*c) - (4*a*b*tan(1/2*d*x + 1/2*c) + b^2)/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 0.33, size = 105, normalized size = 3.09

$$\frac{2\,a^2\,\mathrm{atan}\left(\frac{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{b^2\cot(c+d\,x)}{d} + \frac{2\,a\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sin(c + d*x))^2,x)
```

```
[Out] (2*a^2*atan((a*cos(c/2 + (d*x)/2) + 2*b*sin(c/2 + (d*x)/2))/(2*b*cos(c/2 +
(d*x)/2) - a*sin(c/2 + (d*x)/2))))/d - (b^2*cot(c + d*x))/d + (2*a*b*log(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.39 $\int \frac{\csc^5(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=112

$$\frac{a(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^4}-\frac{2a^4\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}-\frac{(3a^2+2b^2)\cot(x)}{3b^3}+\frac{a\cot(x)\csc(x)}{2b^2}-\frac{\cot(x)\csc^2(x)}{3b}$$

```
[Out] 1/2*a*(2*a^2+b^2)*arctanh(cos(x))/b^4-1/3*(3*a^2+2*b^2)*cot(x)/b^3+1/2*a*co
t(x)*csc(x)/b^2-1/3*cot(x)*csc(x)^2/b-2*a^4*arctanh((a+b*tan(1/2*x))/(a^2-b
^2)^(1/2))/b^4/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3936, 4177, 4167, 4083, 3855, 3916, 2739, 632, 212}

$$\frac{a(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^4}-\frac{(3a^2+2b^2)\cot(x)}{3b^3}-\frac{2a^4\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}+\frac{a\cot(x)\csc(x)}{2b^2}-\frac{\cot(x)\csc^2(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^5/(a + b*Csc[x]),x]
```

```
[Out] (a*(2*a^2 + b^2)*ArcTanh[Cos[x]])/(2*b^4) - (2*a^4*ArcTanh[(a + b*Tan[x/2])
/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2]) - ((3*a^2 + 2*b^2)*Cot[x])/(3*b^3)
 + (a*Cot[x]*Csc[x])/(2*b^2) - (Cot[x]*Csc[x]^2)/(3*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3936

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Dist[d^3/(b*(n - 2)), Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a*
(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ
[n, 3]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4177

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(
m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^5(x)}{a+b\csc(x)}\,dx = -\frac{\cot(x)\csc^2(x)}{3b}+\frac{\int \frac{\csc^2(x)(2a+2b\csc(x)-3a\csc^2(x))}{a+b\csc(x)}\,dx}{3b}$$
$$= \frac{a\cot(x)\csc(x)}{2b^2}-\frac{\cot(x)\csc^2(x)}{3b}+\frac{\int \frac{\csc(x)(-3a^2+ab\csc(x)+2(3a^2+2b^2)\csc^2(x))}{a+b\csc(x)}\,dx}{6b^2}$$
$$= -\frac{(3a^2+2b^2)\cot(x)}{3b^3}+\frac{a\cot(x)\csc(x)}{2b^2}-\frac{\cot(x)\csc^2(x)}{3b}+\frac{\int \frac{\csc(x)(-3a^2b-3a(2a^2+b^2)\csc(x))}{a+b\csc(x)}}{6b^3}$$
$$= -\frac{(3a^2+2b^2)\cot(x)}{3b^3}+\frac{a\cot(x)\csc(x)}{2b^2}-\frac{\cot(x)\csc^2(x)}{3b}+\frac{a^4\int \frac{\csc(x)}{a+b\csc(x)}\,dx}{b^4}-\frac{(a(2a^2+}{}$$
$$= \frac{a(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^4}-\frac{(3a^2+2b^2)\cot(x)}{3b^3}+\frac{a\cot(x)\csc(x)}{2b^2}-\frac{\cot(x)\csc^2(x)}{3b}+$$
$$= \frac{a(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^4}-\frac{(3a^2+2b^2)\cot(x)}{3b^3}+\frac{a\cot(x)\csc(x)}{2b^2}-\frac{\cot(x)\csc^2(x)}{3b}+$$
$$= \frac{a(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^4}-\frac{(3a^2+2b^2)\cot(x)}{3b^3}+\frac{a\cot(x)\csc(x)}{2b^2}-\frac{\cot(x)\csc^2(x)}{3b}-$$
$$= \frac{a(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^4}-\frac{2a^4\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}-\frac{(3a^2+2b^2)\cot(x)}{3b^3}+\frac{a\cot(}{}$$

Mathematica [A]
time = 1.86, size = 125, normalized size = 1.12

$$\frac{\frac{24a^4\text{ArcTan}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+b(3a^2+2b^2)\cos(3x)\csc^3(x)-3b\cot(x)\csc(x)\left(-2ab+(a^2+2b^2)\csc(x)\right)+6a(2a^2+b^2)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^5/(a + b*Csc[x]),x]
```

```
[Out] ((24*a^4*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + b*(3
*a^2 + 2*b^2)*Cos[3*x]*Csc[x]^3 - 3*b*Cot[x]*Csc[x]*(-2*a*b + (a^2 + 2*b^2)
*Csc[x]) + 6*a*(2*a^2 + b^2)*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(12*b^4)
```

Maple [A]
time = 0.20, size = 156, normalized size = 1.39

method	result

default	$\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)b^2}{3}-ab\left(\tan^2\left(\frac{x}{2}\right)\right)+4a^2\tan\left(\frac{x}{2}\right)+3b^2\tan\left(\frac{x}{2}\right)}{8b^3}-\frac{1}{24b\tan\left(\frac{x}{2}\right)^3}-\frac{4a^2+3b^2}{8b^3\tan\left(\frac{x}{2}\right)}+\frac{a}{8b^2\tan\left(\frac{x}{2}\right)^2}-\frac{a(2a^2+b^2)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2b^4}$
risch	$\frac{i(3iab\,e^{5ix}-6a^2e^{4ix}-3iab\,e^{ix}+12a^2e^{2ix}+12b^2e^{2ix}-6a^2-4b^2)}{3b^3(e^{2ix}-1)^3}+\frac{a^3\ln(e^{ix}+1)}{b^4}+\frac{a\ln(e^{ix}+1)}{2b^2}-\frac{ia^4\ln\left(e^{ix}+\frac{i\left(\sqrt{-a^2+b^2}\ldots\right.}{a\sqrt{-a^2+\ldots}}\right.}{\sqrt{-a^2+b^2}\,b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^5/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b^3*(1/3*tan(1/2*x)^3*b^2-a*b*tan(1/2*x)^2+4*a^2*tan(1/2*x)+3*b^2*tan(1
/2*x))-1/24/b/tan(1/2*x)^3-1/8*(4*a^2+3*b^2)/b^3/tan(1/2*x)+1/8*a/b^2/tan(1
/2*x)^2-1/2/b^4*a*(2*a^2+b^2)*ln(tan(1/2*x))+2/b^4*a^4/(-a^2+b^2)^(1/2)*arc
tan(1/2*(2*b*tan(1/2*x)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(98) = 196.
time = 3.82, size = 607, normalized size = 5.42

$$\left[\frac{4\left(3a^4b-a^2b^3-2b^5\right)\cos(x)^3-6\left(a^4\cos(x)^2-a^4\right)\sqrt{a^2-b^2}\log\left(-\frac{(a^2-2b^2)\cos(x)^2+2ab\sin(x)+a^2+b^2-2(b\cos(x)\sin(x)+a\cos(x))\sqrt{a^2-b^2}}{a^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)\sin(x)+6\left(a^3b^2-ab^4\right)\cos(x)\sin(x)+3\left(2a^5-a^3b^2-ab^4-(2a^5-a^3b^2-ab^4)\cos(x)^2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)-3\left(2a^5-a^3b^2-ab^4-(2a^5-a^3b^2-ab^4)\cos(x)^2\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)-12\left(a^4b-b^5\right)\cos(x)}{12\left(a^2b^4-b^6-(a^2b^4-b^6)\cos(x)^2\right)\sin(x)},\ \frac{4\left(3a^4b-a^2b^3-2b^5\right)\cos(x)^3+12\left(a^4\cos(x)^2-a^4\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\sin(x)+a)}{(a^2-b^2)\cos(x)}\right)\sin(x)+6\left(a^3b^2-ab^4\right)\cos(x)\sin(x)+3\left(2a^5-a^3b^2-ab^4-(2a^5-a^3b^2-ab^4)\cos(x)^2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)-3\left(2a^5-a^3b^2-ab^4-(2a^5-a^3b^2-ab^4)\cos(x)^2\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)-12\left(a^4b-b^5\right)\cos(x)}{12\left(a^2b^4-b^6-(a^2b^4-b^6)\cos(x)^2\right)\sin(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(3*a^4*b - a^2*b^3 - 2*b^5)*cos(x)^3 - 6*(a^4*cos(x)^2 - a^4)*sqrt
(a^2 - b^2)*log(-((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2 + b^2 - 2*(b*
cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*a*b*sin(x) - a
^2 - b^2))*sin(x) + 6*(a^3*b^2 - a*b^4)*cos(x)*sin(x) + 3*(2*a^5 - a^3*b^2
- a*b^4 - (2*a^5 - a^3*b^2 - a*b^4)*cos(x)^2)*log(1/2*cos(x) + 1/2)*sin(x)
- 3*(2*a^5 - a^3*b^2 - a*b^4 - (2*a^5 - a^3*b^2 - a*b^4)*cos(x)^2)*log(-1/2
```

```
*cos(x) + 1/2)*sin(x) - 12*(a^4*b - b^5)*cos(x))/((a^2*b^4 - b^6 - (a^2*b^4
 - b^6)*cos(x)^2)*sin(x)), 1/12*(4*(3*a^4*b - a^2*b^3 - 2*b^5)*cos(x)^3 + 1
2*(a^4*cos(x)^2 - a^4)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(x)
+ a)/((a^2 - b^2)*cos(x)))*sin(x) + 6*(a^3*b^2 - a*b^4)*cos(x)*sin(x) + 3*(
2*a^5 - a^3*b^2 - a*b^4 - (2*a^5 - a^3*b^2 - a*b^4)*cos(x)^2)*log(1/2*cos(x
) + 1/2)*sin(x) - 3*(2*a^5 - a^3*b^2 - a*b^4 - (2*a^5 - a^3*b^2 - a*b^4)*co
s(x)^2)*log(-1/2*cos(x) + 1/2)*sin(x) - 12*(a^4*b - b^5)*cos(x))/((a^2*b^4
- b^6 - (a^2*b^4 - b^6)*cos(x)^2)*sin(x))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(x)}{a + b\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**5/(a+b*csc(x)),x)
```

```
[Out] Integral(csc(x)**5/(a + b*csc(x)), x)
```

Giac [**A**]
time = 0.42, size = 194, normalized size = 1.73

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}x\right)+a}{\sqrt{-a^2+b^2}}\right)\right)a^4}{\sqrt{-a^2+b^2}\,b^4}+\frac{b^2\tan\left(\frac{1}{2}x\right)^3-3\,ab\tan\left(\frac{1}{2}x\right)^2+12\,a^2\tan\left(\frac{1}{2}x\right)+9\,b^2\tan\left(\frac{1}{2}x\right)}{24\,b^3}-\frac{(2\,a^3+ab^2)\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2\,b^4}+\frac{44\,a^3\tan\left(\frac{1}{2}x\right)^3+22\,ab^2\tan\left(\frac{1}{2}x\right)^3-12\,a^2b\tan\left(\frac{1}{2}x\right)^2-9\,b^3\tan\left(\frac{1}{2}x\right)^2+3\,ab^2\tan\left(\frac{1}{2}x\right)-b^3}{24\,b^4\tan\left(\frac{1}{2}x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 +
b^2)))*a^4/(sqrt(-a^2 + b^2)*b^4) + 1/24*(b^2*tan(1/2*x)^3 - 3*a*b*tan(1/2*
x)^2 + 12*a^2*tan(1/2*x) + 9*b^2*tan(1/2*x))/b^3 - 1/2*(2*a^3 + a*b^2)*log(
abs(tan(1/2*x)))/b^4 + 1/24*(44*a^3*tan(1/2*x)^3 + 22*a*b^2*tan(1/2*x)^3 -
12*a^2*b*tan(1/2*x)^2 - 9*b^3*tan(1/2*x)^2 + 3*a*b^2*tan(1/2*x) - b^3)/(b^4
*tan(1/2*x)^3)
```

Mupad [**B**]
time = 0.79, size = 588, normalized size = 5.25

$$-\frac{b^2\left(\frac{3\,a\sin(2\,x)\sqrt{a^2-b^2}}{4}+\frac{3\,a\sin(3\,x)\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sqrt{a^2-b^2}}{8}-\frac{9\,a\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sin(x)\sqrt{a^2-b^2}}{8}\right)+b^3\left(\frac{\cos(3\,x)\sqrt{a^2-b^2}}{2}-\frac{3\cos(x)\sqrt{a^2-b^2}}{2}\right)-b\left(\frac{3\,a^2\cos(x)\sqrt{a^2-b^2}}{4}-\frac{3\,a^2\cos(3\,x)\sqrt{a^2-b^2}}{4}\right)-\frac{9\,a^3\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sin(x)\sqrt{a^2-b^2}}{4}+\frac{3\,a^3\sin(3\,x)\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sqrt{a^2-b^2}}{4}+\frac{a^4\,\mathrm{atan}\left(\frac{a^4\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,8\mathrm{i}-b^4\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+a\,b^3\cos\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+a^3\,b\cos\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,4\mathrm{i}}{-8\sin\left(\frac{x}{2}\right)a^5-4\cos\left(\frac{x}{2}\right)a^4\,b+4\sin\left(\frac{x}{2}\right)a^3\,b^2+\cos\left(\frac{x}{2}\right)a^2\,b^3+2\sin\left(\frac{x}{2}\right)a\,b^4+\cos\left(\frac{x}{2}\right)b^5}\right)\sin(x)\,9\mathrm{i}}{2}-\frac{a^4\,\mathrm{atan}\left(\frac{a^4\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,8\mathrm{i}-b^4\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+a\,b^3\cos\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+a^3\,b\cos\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,4\mathrm{i}}{-8\sin\left(\frac{x}{2}\right)a^5-4\cos\left(\frac{x}{2}\right)a^4\,b+4\sin\left(\frac{x}{2}\right)a^3\,b^2+\cos\left(\frac{x}{2}\right)a^2\,b^3+2\sin\left(\frac{x}{2}\right)a\,b^4+\cos\left(\frac{x}{2}\right)b^5}\right)\sin(3\,x)\,3\mathrm{i}}{2}}{\frac{3\,b^4\sin(3\,x)\sqrt{a^2-b^2}}{4}-\frac{9\,b^4\sin(x)\sqrt{a^2-b^2}}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^5*(a + b/sin(x))),x)
```

```
[Out] -(b^2*((3*a*sin(2*x)*(a^2 - b^2)^(1/2))/4 + (3*a*sin(3*x)*log(sin(x/2)/cos(
x/2))*(a^2 - b^2)^(1/2))/8 - (9*a*log(sin(x/2)/cos(x/2))*sin(x)*(a^2 - b^2)
^(1/2))/8) + b^3*((cos(3*x)*(a^2 - b^2)^(1/2))/2 - (3*cos(x)*(a^2 - b^2)^(1
```

```
/2))/2) - b*((3*a^2*cos(x)*(a^2 - b^2)^(1/2))/4 - (3*a^2*cos(3*x)*(a^2 - b^
2)^(1/2))/4) + (a^4*atan((a^4*sin(x/2)*(a^2 - b^2)^(1/2)*8i - b^4*sin(x/2)*
(a^2 - b^2)^(1/2)*1i + a*b^3*cos(x/2)*(a^2 - b^2)^(1/2)*1i + a^3*b*cos(x/2)
*(a^2 - b^2)^(1/2)*4i)/(b^5*cos(x/2) - 8*a^5*sin(x/2) + a^2*b^3*cos(x/2) +
4*a^3*b^2*sin(x/2) - 4*a^4*b*cos(x/2) + 2*a*b^4*sin(x/2)))*sin(x)*9i)/2 - (
a^4*atan((a^4*sin(x/2)*(a^2 - b^2)^(1/2)*8i - b^4*sin(x/2)*(a^2 - b^2)^(1/2
)*1i + a*b^3*cos(x/2)*(a^2 - b^2)^(1/2)*1i + a^3*b*cos(x/2)*(a^2 - b^2)^(1/
2)*4i)/(b^5*cos(x/2) - 8*a^5*sin(x/2) + a^2*b^3*cos(x/2) + 4*a^3*b^2*sin(x/
2) - 4*a^4*b*cos(x/2) + 2*a*b^4*sin(x/2)))*sin(3*x)*3i)/2 - (9*a^3*log(sin(
x/2)/cos(x/2))*sin(x)*(a^2 - b^2)^(1/2))/4 + (3*a^3*sin(3*x)*log(sin(x/2)/c
os(x/2))*(a^2 - b^2)^(1/2))/4)/((3*b^4*sin(3*x)*(a^2 - b^2)^(1/2))/4 - (9*b
^4*sin(x)*(a^2 - b^2)^(1/2))/4)
```

3.40 $\int \frac{\csc^4(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=84

$$-\frac{(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{a\cot(x)}{b^2}-\frac{\cot(x)\csc(x)}{2b}$$

```
[Out] -1/2*(2*a^2+b^2)*arctanh(cos(x))/b^3+a*cot(x)/b^2-1/2*cot(x)*csc(x)/b+2*a^3
*arctanh((a+b*tan(1/2*x))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.615$, Rules used = {3936, 4167, 4083, 3855, 3916, 2739, 632, 212}

$$-\frac{(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}}+\frac{a\cot(x)}{b^2}-\frac{\cot(x)\csc(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + b*Csc[x]),x]
```

```
[Out] -1/2*((2*a^2 + b^2)*ArcTanh[Cos[x]])/b^3 + (2*a^3*ArcTanh[(a + b*Tan[x/2])/
Sqrt[a^2 - b^2]])/(b^3*Sqrt[a^2 - b^2]) + (a*Cot[x])/b^2 - (Cot[x]*Csc[x])/
(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3936

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Dist[d^3/(b*(n - 2)), Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a*
(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ
[n, 3]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^4(x)}{a+b\csc(x)}\,dx = -\frac{\cot(x)\csc(x)}{2b} + \frac{\int \frac{\csc(x)(a+b\csc(x)-2a\csc^2(x))}{a+b\csc(x)}\,dx}{2b}$$

$$= \frac{a\cot(x)}{b^2} - \frac{\cot(x)\csc(x)}{2b} + \frac{\int \frac{\csc(x)(ab+(2a^2+b^2)\csc(x))}{a+b\csc(x)}\,dx}{2b^2}$$

$$= \frac{a\cot(x)}{b^2} - \frac{\cot(x)\csc(x)}{2b} - \frac{a^3\int \frac{\csc(x)}{a+b\csc(x)}\,dx}{b^3} + \frac{(2a^2+b^2)\int \csc(x)\,dx}{2b^3}$$

$$= -\frac{(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^3} + \frac{a\cot(x)}{b^2} - \frac{\cot(x)\csc(x)}{2b} - \frac{a^3\int \frac{1}{1+\frac{a\sin(x)}{b}}\,dx}{b^4}$$

$$= -\frac{(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^3} + \frac{a\cot(x)}{b^2} - \frac{\cot(x)\csc(x)}{2b} - \frac{(2a^3)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx,\right.}{b^4}$$

$$= -\frac{(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^3} + \frac{a\cot(x)}{b^2} - \frac{\cot(x)\csc(x)}{2b} + \frac{(4a^3)\operatorname{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\right.}{b^4}$$

$$= -\frac{(2a^2+b^2)\tanh^{-1}(\cos(x))}{2b^3} + \frac{2a^3\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}} + \frac{a\cot(x)}{b^2} - \frac{\cot(x)\csc(x)}{2b}$$

Mathematica [A]

time = 0.53, size = 144, normalized size = 1.71

$$\frac{-\frac{16a^3\operatorname{ArcTan}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + 4ab\cot\left(\frac{x}{2}\right) - b^2\csc^2\left(\frac{x}{2}\right) - 8a^2\log\left(\cos\left(\frac{x}{2}\right)\right) - 4b^2\log\left(\cos\left(\frac{x}{2}\right)\right) + 8a^2\log\left(\sin\left(\frac{x}{2}\right)\right) + 4b^2\log\left(\sin\left(\frac{x}{2}\right)\right) + b^2\sec^2\left(\frac{x}{2}\right) - 4ab\tan\left(\frac{x}{2}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + b*Csc[x]),x]
```

```
[Out] ((-16*a^3*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 4*a
*b*Cot[x/2] - b^2*Csc[x/2]^2 - 8*a^2*Log[Cos[x/2]] - 4*b^2*Log[Cos[x/2]] +
8*a^2*Log[Sin[x/2]] + 4*b^2*Log[Sin[x/2]] + b^2*Sec[x/2]^2 - 4*a*b*Tan[x/2]
)/(8*b^3)
```

Maple [A]

time = 0.16, size = 112, normalized size = 1.33

method	result
default	$-\frac{-\frac{b\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+2a\tan\left(\frac{x}{2}\right)}{4b^2} - \frac{1}{8b\tan\left(\frac{x}{2}\right)^2} + \frac{(4a^2+2b^2)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4b^3} + \frac{a}{2b^2\tan\left(\frac{x}{2}\right)} - \frac{2a^3\arctan\left(\frac{2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{b^3\sqrt{-a^2+b^2}}$

risch	$\frac{2ia\,e^{2ix}+b\,e^{3ix}-2ia+b\,e^{ix}}{\left(e^{2ix}-1\right)^2 b^2}+\frac{\ln\left(e^{ix}-1\right)a^2}{b^3}+\frac{\ln\left(e^{ix}-1\right)}{2b}-\frac{\ln\left(e^{ix}+1\right)a^2}{b^3}-\frac{\ln\left(e^{ix}+1\right)}{2b}+\frac{a^3\ln\left(e^{ix}+\frac{ib\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^2*(-1/2*b*tan(1/2*x)^2+2*a*tan(1/2*x))-1/8/b/tan(1/2*x)^2+1/4/b^3*(4
*a^2+2*b^2)*ln(tan(1/2*x))+1/2*a/b^2/tan(1/2*x)-2/b^3*a^3/(-a^2+b^2)^(1/2)*
arctan(1/2*(2*b*tan(1/2*x)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(74) = 148.
time = 3.26, size = 524, normalized size = 6.24

$$\left[\frac{4\left(a^3b-ab^3\right)\cos(x)\sin(x)-2\left(a^3\cos(x)^2-a^3\right)\sqrt{a^2-b^2}\log\left(\frac{\left(a^2-2b^2\right)\cos(x)^2+2ab\sin(x)+a^2+b^2+2\left(b\cos(x)\sin(x)+a\cos(x)\right)\sqrt{a^2-b^2}}{a^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)-2\left(a^2b^2-b^4\right)\cos(x)-\left(2a^4-a^2b^2-b^4-\left(2a^4-a^2b^2-b^4\right)\cos(x)^2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\left(2a^4-a^2b^2-b^4-\left(2a^4-a^2b^2-b^4\right)\cos(x)^2\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{4\left(a^2b^3-b^5-\left(a^2b^3-b^5\right)\cos(x)^2\right)}, \frac{4\left(a^3b-ab^3\right)\cos(x)\sin(x)-4\left(a^3\cos(x)^2-a^3\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\left(b\sin(x)+a\right)}{\left(a^2-b^2\right)\cos(x)}\right)-2\left(a^2b^2-b^4\right)\cos(x)-\left(2a^4-a^2b^2-b^4-\left(2a^4-a^2b^2-b^4\right)\cos(x)^2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\left(2a^4-a^2b^2-b^4-\left(2a^4-a^2b^2-b^4\right)\cos(x)^2\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{4\left(a^2b^3-b^5-\left(a^2b^3-b^5\right)\cos(x)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^3*b - a*b^3)*cos(x)*sin(x) - 2*(a^3*cos(x)^2 - a^3)*sqrt(a^2 - b
^2)*log(((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2 + b^2 + 2*(b*cos(x)*si
n(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)
) - 2*(a^2*b^2 - b^4)*cos(x) - (2*a^4 - a^2*b^2 - b^4 - (2*a^4 - a^2*b^2 -
b^4)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (2*a^4 - a^2*b^2 - b^4 - (2*a^4 - a^
2*b^2 - b^4)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^2*b^3 - b^5 - (a^2*b^3 -
b^5)*cos(x)^2), 1/4*(4*(a^3*b - a*b^3)*cos(x)*sin(x) - 4*(a^3*cos(x)^2 - a^
3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) + a)/((a^2 - b^2)*co
s(x))) - 2*(a^2*b^2 - b^4)*cos(x) - (2*a^4 - a^2*b^2 - b^4 - (2*a^4 - a^2*b
^2 - b^4)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (2*a^4 - a^2*b^2 - b^4 - (2*a^4
 - a^2*b^2 - b^4)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^2*b^3 - b^5 - (a^2*b
^3 - b^5)*cos(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(x)}{a + b\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+b*csc(x)),x)
```

```
[Out] Integral(csc(x)**4/(a + b*csc(x)), x)
```

Giac [A]
time = 0.43, size = 141, normalized size = 1.68

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}x\right)+a}{\sqrt{-a^2+b^2}}\right)\right)a^3}{\sqrt{-a^2+b^2}\,b^3}+\frac{b\tan\left(\frac{1}{2}x\right)^2-4a\tan\left(\frac{1}{2}x\right)}{8b^2}+\frac{(2a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2b^3}-\frac{12a^2\tan\left(\frac{1}{2}x\right)^2+6b^2\tan\left(\frac{1}{2}x\right)^2-4ab\tan\left(\frac{1}{2}x\right)+b^2}{8b^3\tan\left(\frac{1}{2}x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 +
 b^2)))*a^3/(sqrt(-a^2 + b^2)*b^3) + 1/8*(b*tan(1/2*x)^2 - 4*a*tan(1/2*x))/
b^2 + 1/2*(2*a^2 + b^2)*log(abs(tan(1/2*x)))/b^3 - 1/8*(12*a^2*tan(1/2*x)^2
 + 6*b^2*tan(1/2*x)^2 - 4*a*b*tan(1/2*x) + b^2)/(b^3*tan(1/2*x)^2)
```

Mupad [B]
time = 0.58, size = 515, normalized size = 6.13

$$-\frac{b^2\left(\frac{\cos(x)\sqrt{a^2-b^2}}{2}-\frac{\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sqrt{a^2-b^2}}{4}+\frac{\cos(2x)\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sqrt{a^2-b^2}}{4}\right)-\frac{a^2\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sqrt{a^2-b^2}}{2}-\frac{ab\sin(2x)\sqrt{a^2-b^2}}{2}+\frac{a^2\cos(2x)\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sqrt{a^2-b^2}}{2}+a^3\operatorname{atan}\left(\frac{a^4\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,8\mathrm{i}-b^4\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+ab^3\cos\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+a^3b\cos\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,4\mathrm{i}}{-8\sin\left(\frac{x}{2}\right)a^5-4\cos\left(\frac{x}{2}\right)a^4b+4\sin\left(\frac{x}{2}\right)a^3b^2+\cos\left(\frac{x}{2}\right)a^2b^3+2\sin\left(\frac{x}{2}\right)ab^4+\cos\left(\frac{x}{2}\right)b^5}\right)1\mathrm{i}-a^3\cos(2x)\operatorname{atan}\left(\frac{a^4\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,8\mathrm{i}-b^4\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+ab^3\cos\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+a^3b\cos\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,4\mathrm{i}}{-8\sin\left(\frac{x}{2}\right)a^5-4\cos\left(\frac{x}{2}\right)a^4b+4\sin\left(\frac{x}{2}\right)a^3b^2+\cos\left(\frac{x}{2}\right)a^2b^3+2\sin\left(\frac{x}{2}\right)ab^4+\cos\left(\frac{x}{2}\right)b^5}\right)1\mathrm{i}}{\frac{b^3\sqrt{a^2-b^2}}{2}-\frac{b^3\cos(2x)\sqrt{a^2-b^2}}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + b/sin(x))),x)
```

```
[Out] -(a^3*atan((a^4*sin(x/2)*(a^2 - b^2)^(1/2)*8i - b^4*sin(x/2)*(a^2 - b^2)^(1
/2)*1i + a*b^3*cos(x/2)*(a^2 - b^2)^(1/2)*1i + a^3*b*cos(x/2)*(a^2 - b^2)^(
1/2)*4i)/(b^5*cos(x/2) - 8*a^5*sin(x/2) + a^2*b^3*cos(x/2) + 4*a^3*b^2*sin(
x/2) - 4*a^4*b*cos(x/2) + 2*a*b^4*sin(x/2)))*1i + b^2*((cos(x)*(a^2 - b^2)^
(1/2))/2 - (log(sin(x/2)/cos(x/2))*(a^2 - b^2)^(1/2))/4 + (cos(2*x)*log(sin
(x/2)/cos(x/2))*(a^2 - b^2)^(1/2))/4) - (a^2*log(sin(x/2)/cos(x/2))*(a^2 -
b^2)^(1/2))/2 - a^3*cos(2*x)*atan((a^4*sin(x/2)*(a^2 - b^2)^(1/2)*8i - b^4*
sin(x/2)*(a^2 - b^2)^(1/2)*1i + a*b^3*cos(x/2)*(a^2 - b^2)^(1/2)*1i + a^3*b
*cos(x/2)*(a^2 - b^2)^(1/2)*4i)/(b^5*cos(x/2) - 8*a^5*sin(x/2) + a^2*b^3*co
s(x/2) + 4*a^3*b^2*sin(x/2) - 4*a^4*b*cos(x/2) + 2*a*b^4*sin(x/2)))*1i - (a
*b*sin(2*x)*(a^2 - b^2)^(1/2))/2 + (a^2*cos(2*x)*log(sin(x/2)/cos(x/2))*(a^
2 - b^2)^(1/2))/2)/((b^3*(a^2 - b^2)^(1/2))/2 - (b^3*cos(2*x)*(a^2 - b^2)^(
1/2))/2)
```

3.41 $\int \frac{\csc^3(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=62

$$\frac{a\tanh^{-1}(\cos(x))}{b^2}-\frac{2a^2\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cot(x)}{b}$$

```
[Out] a*arctanh(cos(x))/b^2-cot(x)/b-2*a^2*arctanh((a+b*tan(1/2*x))/(a^2-b^2)^(1/
2))/b^2/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3875, 3874, 3855, 3916, 2739, 632, 212}

$$-\frac{2a^2\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}+\frac{a\tanh^{-1}(\cos(x))}{b^2}-\frac{\cot(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Csc[x]),x]
```

```
[Out] (a*ArcTanh[Cos[x]])/b^2 - (2*a^2*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b^2]])
/(b^2*Sqrt[a^2 - b^2]) - Cot[x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(x)}{a+b\csc(x)}\,dx &= -\frac{\cot(x)}{b}-\frac{a\int \frac{\csc^2(x)}{a+b\csc(x)}\,dx}{b}\\
&= -\frac{\cot(x)}{b}-\frac{a\int \csc(x)\,dx}{b^2}+\frac{a^2\int \frac{\csc(x)}{a+b\csc(x)}\,dx}{b^2}\\
&= \frac{a\tanh^{-1}(\cos(x))}{b^2}-\frac{\cot(x)}{b}+\frac{a^2\int \frac{1}{1+\frac{a\sin(x)}{b}}\,dx}{b^3}\\
&= \frac{a\tanh^{-1}(\cos(x))}{b^2}-\frac{\cot(x)}{b}+\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{a\tanh^{-1}(\cos(x))}{b^2}-\frac{\cot(x)}{b}-\frac{(4a^2)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}+2\tan\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{a\tanh^{-1}(\cos(x))}{b^2}-\frac{2a^2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cot(x)}{b}
\end{aligned}
$$

Mathematica [A]

time = 0.22, size = 106, normalized size = 1.71

$$\frac{\csc\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\left(2a^2\mathrm{ArcTan}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)\sin(x)+\sqrt{-a^2+b^2}\left(-b\cos(x)+a\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)\right)\right)}{2b^2\sqrt{-a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Csc[x]),x]
```

```
[Out] (Csc[x/2]*Sec[x/2]*(2*a^2*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]]*Sin[x]
+ Sqrt[-a^2 + b^2]*(-(b*Cos[x]) + a*(Log[Cos[x/2]] - Log[Sin[x/2]])*Sin[x])
))/(2*b^2*Sqrt[-a^2 + b^2])
```

Maple [A]
time = 0.14, size = 77, normalized size = 1.24

method	result
default	$\frac{\tan\left(\frac{x}{2}\right)}{2b}-\frac{1}{2b\tan\left(\frac{x}{2}\right)}-\frac{a\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b^2}+\frac{2a^2\arctan\left(\frac{2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{b^2\sqrt{-a^2+b^2}}$
risch	$-\frac{2i}{b\left(\mathrm{e}^{2ix}-1\right)}+\frac{a\ln\left(\mathrm{e}^{ix}+1\right)}{b^2}-\frac{a\ln\left(\mathrm{e}^{ix}-1\right)}{b^2}+\frac{ia^2\ln\left(\mathrm{e}^{ix}+\frac{i\left(\sqrt{-a^2+b^2}\,b-a^2+b^2\right)}{a\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^2}-\frac{ia^2\ln\left(\mathrm{e}^{ix}+\frac{i\left(\sqrt{-a^2+b^2}\,b+a\right.}{a\sqrt{-a^2+b^2}}\right.}{\sqrt{-a^2+b^2}\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*tan(1/2*x)-1/2/b/tan(1/2*x)-a/b^2*ln(tan(1/2*x))+2*a^2/b^2/(-a^2+b^2)
^(1/2)*arctan(1/2*(2*b*tan(1/2*x)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(56) = 112.
time = 2.75, size = 308, normalized size = 4.97

$$\left[\frac{\sqrt{a^2-b^2}\,a^2\log\left(-\frac{\left(a^2-2b^2\right)\cos(x)^2+2ab\sin(x)+a^2+b^2-2\left(b\cos(x)\sin(x)+a\cos(x)\right)\sqrt{a^2-b^2}}{a^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)\sin(x)+\left(a^3-ab^2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)-\left(a^3-ab^2\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)-2\left(a^2b-b^3\right)\cos(x)}{2\left(a^2b^2-b^4\right)\sin(x)},\ -\frac{2\sqrt{-a^2+b^2}\,a^2\arctan\left(-\frac{\sqrt{-a^2+b^2}\left(b\sin(x)+a\right)}{\left(a^2-b^2\right)\cos(x)}\right)\sin(x)-\left(a^3-ab^2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+\left(a^3-ab^2\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+2\left(a^2b-b^3\right)\cos(x)}{2\left(a^2b^2-b^4\right)\sin(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*a^2*log(-((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2
 + b^2 - 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*
a*b*sin(x) - a^2 - b^2))*sin(x) + (a^3 - a*b^2)*log(1/2*cos(x) + 1/2)*sin(x
) - (a^3 - a*b^2)*log(-1/2*cos(x) + 1/2)*sin(x) - 2*(a^2*b - b^3)*cos(x))/(
(a^2*b^2 - b^4)*sin(x)), -1/2*(2*sqrt(-a^2 + b^2)*a^2*arctan(-sqrt(-a^2 + b
^2)*(b*sin(x) + a)/((a^2 - b^2)*cos(x)))*sin(x) - (a^3 - a*b^2)*log(1/2*cos
(x) + 1/2)*sin(x) + (a^3 - a*b^2)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(a^2*b
- b^3)*cos(x))/((a^2*b^2 - b^4)*sin(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{a + b\csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*csc(x)),x)
```

```
[Out] Integral(csc(x)**3/(a + b*csc(x)), x)
```

Giac [A]
time = 0.42, size = 98, normalized size = 1.58

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(b) + \arctan\left(\frac{b\tan\left(\frac{1}{2}x\right)+a}{\sqrt{-a^2+b^2}}\right)\right)a^2}{\sqrt{-a^2+b^2}\,b^2} - \frac{a\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{b^2} + \frac{\tan\left(\frac{1}{2}x\right)}{2b} + \frac{2a\tan\left(\frac{1}{2}x\right) - b}{2b^2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 +
b^2)))*a^2/(sqrt(-a^2 + b^2)*b^2) - a*log(abs(tan(1/2*x)))/b^2 + 1/2*tan(1/
2*x)/b + 1/2*(2*a*tan(1/2*x) - b)/(b^2*tan(1/2*x))
```

Mupad [B]
time = 0.49, size = 135, normalized size = 2.18

$$-\frac{1}{b\tan(x)} - \frac{a\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b^2} - \frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,4\mathrm{i} - b^2\tan\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i} + a\,b\sqrt{a^2-b^2}\,2\mathrm{i}}{4\tan\left(\frac{x}{2}\right)a^3 + 2a^2b - 3\tan\left(\frac{x}{2}\right)a\,b^2 - b^3}\right)2\mathrm{i}}{b^2\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + b/sin(x))),x)
```

```
[Out] - 1/(b*tan(x)) - (a*log(tan(x/2)))/b^2 - (a^2*atan((a^2*tan(x/2)*(a^2 - b^2
)^(1/2)*4i - b^2*tan(x/2)*(a^2 - b^2)^(1/2)*1i + a*b*(a^2 - b^2)^(1/2)*2i)/
(4*a^3*tan(x/2) + 2*a^2*b - b^3 - 3*a*b^2*tan(x/2)))*2i)/(b^2*(a^2 - b^2)^(
1/2))
```

3.42 $\int \frac{\csc^2(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}(\cos(x))}{b}+\frac{2a\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}$$

```
[Out] -arctanh(cos(x))/b+2*a*arctanh((a+b*tan(1/2*x))/(a^2-b^2)^(1/2))/b/(a^2-b^2
)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3874, 3855, 3916, 2739, 632, 212}

$$\frac{2a\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Csc[x]),x]
```

```
[Out] -(ArcTanh[Cos[x]]/b) + (2*a*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*S
qrt[a^2 - b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(x)}{a+b\csc(x)}\,dx &= \frac{\int \csc(x)\,dx}{b} - \frac{a\int \frac{\csc(x)}{a+b\csc(x)}\,dx}{b}\\
&= -\frac{\tanh^{-1}(\cos(x))}{b} - \frac{a\int \frac{1}{1+\frac{a\sin(x)}{b}}\,dx}{b^2}\\
&= -\frac{\tanh^{-1}(\cos(x))}{b} - \frac{(2a)\mathrm{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{\tanh^{-1}(\cos(x))}{b} + \frac{(4a)\mathrm{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}+2\tan\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{\tanh^{-1}(\cos(x))}{b} + \frac{2a\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 62, normalized size = 1.17

$$\frac{-\frac{2a\mathrm{ArcTan}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Csc[x]),x]
```

```
[Out] ((-2*a*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - Log[Co
s[x/2]] + Log[Sin[x/2]])/b
```

Maple [A]
time = 0.11, size = 53, normalized size = 1.00

method	result	size
default	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b}-\frac{2a\arctan\left(\frac{2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{b\sqrt{-a^{2}+b^{2}}}$	53
risch	$-\frac{a\ln\left(\mathrm{e}^{ix}+\frac{ib\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{\sqrt{a^{2}-b^{2}}\,a}\right)}{\sqrt{a^{2}-b^{2}}\,b}+\frac{a\ln\left(\mathrm{e}^{ix}+\frac{ib\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{\sqrt{a^{2}-b^{2}}\,a}\right)}{\sqrt{a^{2}-b^{2}}\,b}+\frac{\ln\left(\mathrm{e}^{ix}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{ix}+1\right)}{b}$	152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*ln(tan(1/2*x))-2*a/b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*x)+2*a)/(
-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(47) = 94$.
time = 2.79, size = 245, normalized size = 4.62

$$\left[\frac{\sqrt{a^{2}-b^{2}}\,a\log\left(\frac{(a^{2}-2\,b^{2})\cos(x)^{2}+2\,ab\sin(x)+a^{2}+b^{2}+2\,(b\cos(x)\sin(x)+a\cos(x))\sqrt{a^{2}-b^{2}}}{a^{2}\cos(x)^{2}-2\,ab\sin(x)-a^{2}-b^{2}}\right)-(a^{2}-b^{2})\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+(a^{2}-b^{2})\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2\,(a^{2}b-b^{3})},\ \frac{2\sqrt{-a^{2}+b^{2}}\,a\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}\,(b\sin(x)+a)}{(a^{2}-b^{2})\cos(x)}\right)-(a^{2}-b^{2})\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+(a^{2}-b^{2})\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2\,(a^{2}b-b^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*a*log(((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2 +
b^2 + 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*a*b
*sin(x) - a^2 - b^2)) - (a^2 - b^2)*log(1/2*cos(x) + 1/2) + (a^2 - b^2)*log
```

```
(-1/2*cos(x) + 1/2))/(a^2*b - b^3), 1/2*(2*sqrt(-a^2 + b^2)*a*arctan(-sqrt(
-a^2 + b^2)*(b*sin(x) + a)/((a^2 - b^2)*cos(x))) - (a^2 - b^2)*log(1/2*cos(
x) + 1/2) + (a^2 - b^2)*log(-1/2*cos(x) + 1/2))/(a^2*b - b^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a+b\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*csc(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*csc(x)), x)
```

Giac [**A**]
time = 0.42, size = 63, normalized size = 1.19

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}x\right)+a}{\sqrt{-a^2+b^2}}\right)\right)a}{\sqrt{-a^2+b^2}\,b}+\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 +
 b^2)))*a/(sqrt(-a^2 + b^2)*b) + log(abs(tan(1/2*x)))/b
```

Mupad [**B**]
time = 0.43, size = 129, normalized size = 2.43

$$\frac{\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{a^2-b^2}\,\left(4\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{x}{2}\right)\,a\,b-1\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,b^2\right)}{a^3\,\sin\left(\frac{x}{2}\right)\,4\mathrm{i}+b\,\cos\left(\frac{x}{2}\right)\,\left(a^2-b^2\right)\,1\mathrm{i}+a^2\,b\,\cos\left(\frac{x}{2}\right)\,1\mathrm{i}-a\,b^2\,\sin\left(\frac{x}{2}\right)\,3\mathrm{i}}\right)}{b\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b/sin(x))),x)
```

```
[Out] log(sin(x/2)/cos(x/2))/b - (2*a*atanh(((a^2 - b^2)^(1/2)*(a^2*sin(x/2)*4i -
 b^2*sin(x/2)*1i + a*b*cos(x/2)*2i))/(a^3*sin(x/2)*4i + b*cos(x/2)*(a^2 - b
^2)*1i + a^2*b*cos(x/2)*1i - a*b^2*sin(x/2)*3i)))/(b*(a^2 - b^2)^(1/2))
```

3.43 $\int \frac{\csc(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{2\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] -2*arctanh((a+b*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3916, 2739, 632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Csc[x]),x]
```

```
[Out] (-2*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc(x)}{a+b\csc(x)}\,dx = \frac{\int \frac{1}{1+\frac{a\sin(x)}{b}}\,dx}{b}$$
$$= \frac{2\mathrm{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b}$$
$$= -\frac{4\mathrm{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}+2\tan\left(\frac{x}{2}\right)\right)}{b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$\frac{2\mathrm{ArcTan}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Csc[x]),x]
[Out] (2*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

Maple [A]
time = 0.06, size = 39, normalized size = 0.98

method	result	size
default	$\frac{2\arctan\left(\frac{2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	39
risch	$-\frac{i\ln\left(e^{ix}+\frac{i\left(\sqrt{-a^2+b^2}\,b+a^2-b^2\right)}{a\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{i\ln\left(e^{ix}+\frac{i\left(\sqrt{-a^2+b^2}\,b-a^2+b^2\right)}{a\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*x)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 2.94, size = 154, normalized size = 3.85

$$\left[\frac{\log\left(-\frac{(a^2-2\,b^2)\cos(x)^2+2\,ab\sin(x)+a^2+b^2-2\,(b\cos(x)\sin(x)+a\cos(x))\sqrt{a^2-b^2}}{a^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)}{2\,\sqrt{a^2-b^2}},\ -\frac{\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\sin(x)+a)}{(a^2-b^2)\cos(x)}\right)}{a^2-b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2 + b^2 - 2*(b*cos(x)*
sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^
2))/sqrt(a^2 - b^2), -sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) +
 a)/((a^2 - b^2)*cos(x)))/(a^2 - b^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{a+b\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*csc(x)),x)
```

```
[Out] Integral(csc(x)/(a + b*csc(x)), x)
```

Giac [**A**]
time = 0.41, size = 48, normalized size = 1.20

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}\,x\right)+a}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 +
b^2)))/sqrt(-a^2 + b^2)
```

Mupad [B]
time = 0.29, size = 36, normalized size = 0.90

$$-\frac{2\,\mathrm{atanh}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + b/sin(x))),x)
```

```
[Out] -(2*atanh((a + b*tan(x/2))/((a + b)^(1/2)*(a - b)^(1/2))))/((a + b)^(1/2)*(
a - b)^(1/2))
```

3.44 $\int \frac{1}{a+b\csc(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{x}{a}+\frac{2b\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}$$

```
[Out] x/a+2*b*arctanh((a+b*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/d/(a^2-b^2)^(1/
2)
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3868, 2739, 632, 212}

$$\frac{2b\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csc[c + d*x])^(-1),x]
```

```
[Out] x/a + (2*b*ArcTanh[(a + b*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 -
 b^2]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\csc(c+dx)}\,dx &= \frac{x}{a}-\frac{\int \frac{1}{1+\frac{a\sin(c+dx)}{b}}\,dx}{a}\\ &= \frac{x}{a}-\frac{2\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\\ &= \frac{x}{a}+\frac{4\text{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}+2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\\ &= \frac{x}{a}+\frac{2b\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 59, normalized size = 1.04

$$\frac{\frac{c}{d}+x-\frac{2b\text{ArcTan}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csc[c + d*x])^(-1),x]
[Out] (c/d + x - (2*b*ArcTan[(a + b*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a
^2 + b^2]*d))/a
```

Maple [A]
time = 0.10, size = 68, normalized size = 1.19

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\arctan\left(\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}}{d}$	68

default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\arctan\left(\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}}{d}$	68
risch	$\frac{x}{a}+\frac{b\ln\left(e^{i(dx+c)}+\frac{ib\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,da}-\frac{b\ln\left(e^{i(dx+c)}+\frac{ib\sqrt{a^2-b^2}-a^2+b^2}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,da}$	146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csc(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a*arctan(tan(1/2*d*x+1/2*c))-2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*
tan(1/2*d*x+1/2*c)+2*a)/(-a^2+b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.82, size = 238, normalized size = 4.18

$$\left[\frac{2\,(a^2-b^2)dx+\sqrt{a^2-b^2}\,b\log\left(\frac{(a^2-2\,b^2)\cos(dx+c)^2+2\,ab\sin(dx+c)+a^2+b^2+2\,(b\cos(dx+c)\sin(dx+c)+a\cos(dx+c))\sqrt{a^2-b^2}}{a^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)}{2\,(a^3-ab^2)d},\ \frac{(a^2-b^2)dx+\sqrt{-a^2+b^2}\,b\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\sin(dx+c)+a)}{(a^2-b^2)\cos(dx+c)}\right)}{(a^3-ab^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*x + sqrt(a^2 - b^2)*b*log(((a^2 - 2*b^2)*cos(d*x + c)
^2 + 2*a*b*sin(d*x + c) + a^2 + b^2 + 2*(b*cos(d*x + c)*sin(d*x + c) + a*co
s(d*x + c))*sqrt(a^2 - b^2))/(a^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2)))/((a^3 - a*b^2)*d), ((a^2 - b^2)*d*x + sqrt(-a^2 + b^2)*b*arctan(-
sqrt(-a^2 + b^2)*(b*sin(d*x + c) + a)/((a^2 - b^2)*cos(d*x + c))))/((a^3 -
a*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*csc(c + d*x)), x)
```

Giac [A]
time = 0.43, size = 77, normalized size = 1.35

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}\,a}-\frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*d*x + 1/2*
c) + a)/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2)*a) - (d*x + c)/a)/d
```

Mupad [B]
time = 0.45, size = 184, normalized size = 3.23

$$\frac{x}{a}-\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)-2\,b^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)+a\,b^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,a^2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{a\,\left(2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\right)\,\sqrt{a^2-b^2}}\right)}{a\,d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/sin(c + d*x)),x)
```

```
[Out] x/a - (2*b*atanh((2*a^2*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 2*b^4*sin(c/2 + (d
*x)/2) - 2*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2) + a*b^3*cos(c/2 + (d*x)/2) +
3*a^2*b^2*sin(c/2 + (d*x)/2) + a*b*cos(c/2 + (d*x)/2)*(a^2 - b^2))/(a*(2*a^
2*sin(c/2 + (d*x)/2) + a*b*cos(c/2 + (d*x)/2))*(a^2 - b^2)^(1/2))))/(a*d*(a
^2 - b^2)^(1/2))
```

3.45 $\int \frac{\sin(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=61

$$-\frac{bx}{a^2}-\frac{2b^2\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{a}$$

```
[Out] -b*x/a^2-cos(x)/a-2*b^2*arctanh((a+b*tan(1/2*x))/(a^2-b^2)^(1/2))/a^2/(a^2-
b^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {3938, 12, 3868, 2739, 632, 212}

$$-\frac{2b^2\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}-\frac{bx}{a^2}-\frac{\cos(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Csc[x]),x]
```

```
[Out] -((b*x)/a^2) - (2*b^2*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[
a^2 - b^2]) - Cos[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{a+b\csc(x)}\,dx &= -\frac{\cos(x)}{a}-\frac{\int \frac{b}{a+b\csc(x)}\,dx}{a}\\
&= -\frac{\cos(x)}{a}-\frac{b\int \frac{1}{a+b\csc(x)}\,dx}{a}\\
&= -\frac{bx}{a^2}-\frac{\cos(x)}{a}+\frac{b\int \frac{1}{1+\frac{a\sin(x)}{b}}\,dx}{a^2}\\
&= -\frac{bx}{a^2}-\frac{\cos(x)}{a}+\frac{(2b)\mathrm{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{bx}{a^2}-\frac{\cos(x)}{a}-\frac{(4b)\mathrm{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}+2\tan\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{bx}{a^2}-\frac{2b^2\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 0.92

$$-\frac{bx-\frac{2b^2\mathrm{ArcTan}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+a\cos(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Csc[x]),x]
```

```
[Out] -((b*x - (2*b^2*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
 + a*Cos[x])/a^2)
```

Maple [A]
time = 0.09, size = 73, normalized size = 1.20

method	result	size
default	$\frac{-\frac{2a}{\tan^2\left(\frac{x}{2}\right)+1}-2b\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}+\frac{2b^2\arctan\left(\frac{2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}}$	73
risch	$-\frac{xb}{a^2}-\frac{e^{ix}}{2a}-\frac{e^{-ix}}{2a}+\frac{ib^2\ln\left(e^{ix}+\frac{i\left(\sqrt{-a^2+b^2}\,b-a^2+b^2\right)}{a\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^2}-\frac{ib^2\ln\left(e^{ix}+\frac{i\left(\sqrt{-a^2+b^2}\,b+a^2-b^2\right)}{a\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^2}$	161

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^2*(-a/(tan(1/2*x)^2+1)-b*arctan(tan(1/2*x)))+2*b^2/a^2/(-a^2+b^2)^(1/2)
*arctan(1/2*(2*b*tan(1/2*x)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.16, size = 235, normalized size = 3.85

$$\left[\frac{\sqrt{a^2-b^2}\,b^2\log\left(-\frac{(a^2-2\,b^2)\cos(x)^2+2\,ab\sin(x)+a^2+b^2-2\,(b\cos(x)\sin(x)+a\cos(x))\sqrt{a^2-b^2}}{a^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)-2\,(a^2b-b^3)x-2\,(a^3-ab^2)\cos(x)}{2\,(a^4-a^2b^2)},-\frac{\sqrt{-a^2+b^2}\,b^2\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\sin(x)+a)}{(a^2-b^2)\cos(x)}\right)+(a^2b-b^3)x+(a^3-ab^2)\cos(x)}{a^4-a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*b^2*log(-((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2
 + b^2 - 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*
a*b*sin(x) - a^2 - b^2)) - 2*(a^2*b - b^3)*x - 2*(a^3 - a*b^2)*cos(x))/(a^4
 - a^2*b^2), -(sqrt(-a^2 + b^2)*b^2*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) + a)
/((a^2 - b^2)*cos(x))) + (a^2*b - b^3)*x + (a^3 - a*b^2)*cos(x))/(a^4 - a^2
*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{a + b\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*csc(x)),x)
```

```
[Out] Integral(sin(x)/(a + b*csc(x)), x)
```

Giac [A]
time = 0.40, size = 77, normalized size = 1.26

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}x\right)+a}{\sqrt{-a^2+b^2}}\right)\right)b^2}{\sqrt{-a^2+b^2}\,a^2}-\frac{bx}{a^2}-\frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 +
b^2)))*b^2/(sqrt(-a^2 + b^2)*a^2) - b*x/a^2 - 2/((tan(1/2*x)^2 + 1)*a)
```

Mupad [B]
time = 0.65, size = 766, normalized size = 12.56

$$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)^2+1\right)}-\frac{b\,x}{a^2}-\frac{b^2\,\mathrm{atan}\left(\frac{\frac{b^2\sqrt{a^2-b^2}\left(\frac{32b^4}{a}-\frac{32\tan\left(\frac{x}{2}\right)\left(2ab^5-2a^3b^3\right)}{a^3}+\frac{b^2\sqrt{a^2-b^2}\left(32a^2b^2+64ab^3\tan\left(\frac{x}{2}\right)+\frac{b^2\sqrt{a^2-b^2}\left(32a^3b^2+\frac{32\tan\left(\frac{x}{2}\right)\left(3a^7b-2a^5b^3\right)}{a^3}\right)}{a^4-a^2b^2}\right)}{a^4-a^2b^2}\right)1i}{a^4-a^2b^2}-\frac{b^2\sqrt{a^2-b^2}\left(\frac{32\tan\left(\frac{x}{2}\right)\left(2ab^5-2a^3b^3\right)}{a^3}-\frac{32b^4}{a}+\frac{b^2\sqrt{a^2-b^2}\left(32a^2b^2+64ab^3\tan\left(\frac{x}{2}\right)-\frac{b^2\sqrt{a^2-b^2}\left(32a^3b^2+\frac{32\tan\left(\frac{x}{2}\right)\left(3a^7b-2a^5b^3\right)}{a^3}\right)}{a^4-a^2b^2}\right)}{a^4-a^2b^2}\right)1i}{a^4-a^2b^2}}{\frac{128b^5\tan\left(\frac{x}{2}\right)}{a^3}+\frac{b^2\sqrt{a^2-b^2}\left(\frac{32b^4}{a}-\frac{32\tan\left(\frac{x}{2}\right)\left(2ab^5-2a^3b^3\right)}{a^3}+\frac{b^2\sqrt{a^2-b^2}\left(32a^2b^2+64ab^3\tan\left(\frac{x}{2}\right)+\frac{b^2\sqrt{a^2-b^2}\left(32a^3b^2+\frac{32\tan\left(\frac{x}{2}\right)\left(3a^7b-2a^5b^3\right)}{a^3}\right)}{a^4-a^2b^2}\right)}{a^4-a^2b^2}\right)}{a^4-a^2b^2}+\frac{b^2\sqrt{a^2-b^2}\left(\frac{32\tan\left(\frac{x}{2}\right)\left(2ab^5-2a^3b^3\right)}{a^3}-\frac{32b^4}{a}+\frac{b^2\sqrt{a^2-b^2}\left(32a^2b^2+64ab^3\tan\left(\frac{x}{2}\right)-\frac{b^2\sqrt{a^2-b^2}\left(32a^3b^2+\frac{32\tan\left(\frac{x}{2}\right)\left(3a^7b-2a^5b^3\right)}{a^3}\right)}{a^4-a^2b^2}\right)}{a^4-a^2b^2}\right)}{a^4-a^2b^2}}\right)\sqrt{a^2-b^2}\,2i}{a^4-a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b/sin(x)),x)
```

```
[Out] - 2/(a*(tan(x/2)^2 + 1)) - (b*x)/a^2 - (b^2*atan(((b^2*(a^2 - b^2)^(1/2)*((
32*b^4)/a - (32*tan(x/2)*(2*a*b^5 - 2*a^3*b^3))/a^3 + (b^2*(a^2 - b^2)^(1/2
)*(32*a^2*b^2 + 64*a*b^3*tan(x/2) + (b^2*(a^2 - b^2)^(1/2)*(32*a^3*b^2 + (3
```

```
2*tan(x/2)*(3*a^7*b - 2*a^5*b^3))/a^3))/(a^4 - a^2*b^2)))/(a^4 - a^2*b^2))*
1i)/(a^4 - a^2*b^2) - (b^2*(a^2 - b^2)^(1/2)*((32*tan(x/2)*(2*a*b^5 - 2*a^3
*b^3))/a^3 - (32*b^4)/a + (b^2*(a^2 - b^2)^(1/2)*(32*a^2*b^2 + 64*a*b^3*tan
(x/2) - (b^2*(a^2 - b^2)^(1/2)*(32*a^3*b^2 + (32*tan(x/2)*(3*a^7*b - 2*a^5*
b^3))/a^3))/(a^4 - a^2*b^2)))/(a^4 - a^2*b^2))*1i)/(a^4 - a^2*b^2))/((128*b
^5*tan(x/2))/a^3 + (b^2*(a^2 - b^2)^(1/2)*((32*b^4)/a - (32*tan(x/2)*(2*a*b
^5 - 2*a^3*b^3))/a^3 + (b^2*(a^2 - b^2)^(1/2)*(32*a^2*b^2 + 64*a*b^3*tan(x/
2) + (b^2*(a^2 - b^2)^(1/2)*(32*a^3*b^2 + (32*tan(x/2)*(3*a^7*b - 2*a^5*b^3
))/a^3))/(a^4 - a^2*b^2)))/(a^4 - a^2*b^2)))/(a^4 - a^2*b^2) + (b^2*(a^2 -
b^2)^(1/2)*((32*tan(x/2)*(2*a*b^5 - 2*a^3*b^3))/a^3 - (32*b^4)/a + (b^2*(a^
2 - b^2)^(1/2)*(32*a^2*b^2 + 64*a*b^3*tan(x/2) - (b^2*(a^2 - b^2)^(1/2)*(32
*a^3*b^2 + (32*tan(x/2)*(3*a^7*b - 2*a^5*b^3))/a^3))/(a^4 - a^2*b^2)))/(a^4
 - a^2*b^2)))/(a^4 - a^2*b^2)))*(a^2 - b^2)^(1/2)*2i)/(a^4 - a^2*b^2)
```

3.46 $\int \frac{\sin^2(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=82

$$\frac{(a^2+2b^2)\,x}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}+\frac{b\cos(x)}{a^2}-\frac{\cos(x)\sin(x)}{2a}$$

```
[Out] 1/2*(a^2+2*b^2)*x/a^3+b*cos(x)/a^2-1/2*cos(x)*sin(x)/a+2*b^3*arctanh((a+b*t
an(1/2*x))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3938, 4189, 4004, 3916, 2739, 632, 212}

$$\frac{b\cos(x)}{a^2}+\frac{x(a^2+2b^2)}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}-\frac{\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Csc[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) + (2*b^3*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b^2]
])/(a^3*Sqrt[a^2 - b^2]) + (b*Cos[x])/a^2 - (Cos[x]*Sin[x])/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sin^2(x)}{a+b\csc(x)}\,dx = -\frac{\cos(x)\sin(x)}{2a}+\frac{\int \frac{(-2b+a\csc(x)+b\csc^2(x))\sin(x)}{a+b\csc(x)}\,dx}{2a}$$
$$=\frac{b\cos(x)}{a^2}-\frac{\cos(x)\sin(x)}{2a}-\frac{\int \frac{-a^2-2b^2-ab\csc(x)}{a+b\csc(x)}\,dx}{2a^2}$$
$$=\frac{(a^2+2b^2)\,x}{2a^3}+\frac{b\cos(x)}{a^2}-\frac{\cos(x)\sin(x)}{2a}-\frac{b^3\int \frac{\csc(x)}{a+b\csc(x)}\,dx}{a^3}$$
$$=\frac{(a^2+2b^2)\,x}{2a^3}+\frac{b\cos(x)}{a^2}-\frac{\cos(x)\sin(x)}{2a}-\frac{b^2\int \frac{1}{1+\frac{a\sin(x)}{b}}\,dx}{a^3}$$
$$=\frac{(a^2+2b^2)\,x}{2a^3}+\frac{b\cos(x)}{a^2}-\frac{\cos(x)\sin(x)}{2a}-\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{a^3}$$
$$=\frac{(a^2+2b^2)\,x}{2a^3}+\frac{b\cos(x)}{a^2}-\frac{\cos(x)\sin(x)}{2a}+\frac{(4b^2)\,\text{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}+2\tan\left(\right.\right.}{a^3}$$
$$=\frac{(a^2+2b^2)\,x}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}}+\frac{b\cos(x)}{a^2}-\frac{\cos(x)\sin(x)}{2a}$$

Mathematica [A]
time = 0.13, size = 78, normalized size = 0.95

$$\frac{2a^2x+4b^2x-\frac{8b^3\,\text{ArcTan}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+4ab\cos(x)-a^2\sin(2x)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Csc[x]),x]
```

```
[Out] (2*a^2*x + 4*b^2*x - (8*b^3*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt
[-a^2 + b^2] + 4*a*b*Cos[x] - a^2*Sin[2*x])/(4*a^3)
```

Maple [A]
time = 0.10, size = 112, normalized size = 1.37

method	result	size
default	$\frac{\frac{2\left(\frac{a^2\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{a^2\tan\left(\frac{x}{2}\right)}{2}+ab\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^2}+(a^2+2b^2)\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{2b^3\arctan\left(\frac{2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a^3\sqrt{-a^2+b^2}}$	112

risch	$\frac{x}{2a}+\frac{x\,b^2}{a^3}+\frac{b\,\mathrm{e}^{ix}}{2a^2}+\frac{b\,\mathrm{e}^{-ix}}{2a^2}-\frac{b^3\ln\left(\mathrm{e}^{ix}+\frac{ib\sqrt{a^2-b^2}-a^2+b^2}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,a^3}+\frac{b^3\ln\left(\mathrm{e}^{ix}+\frac{ib\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,a^3}-\frac{\sin(2x)}{4a}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^3*((1/2*a^2*tan(1/2*x)^3+a*b*tan(1/2*x)^2-1/2*a^2*tan(1/2*x)+a*b)/(tan(
1/2*x)^2+1)^2+1/2*(a^2+2*b^2)*arctan(tan(1/2*x)))-2*b^3/a^3/(-a^2+b^2)^(1/2
)*arctan(1/2*(2*b*tan(1/2*x)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.41, size = 285, normalized size = 3.48

$$\left[\frac{\sqrt{a^2-b^2}\,b^3\log\left(\frac{(a^2-2\,b^2)\cos(x)^2+2\,ab\sin(x)+a^2+b^2+2\,(b\cos(x)\sin(x)+a\cos(x))\sqrt{a^2-b^2}}{a^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)-(a^4-a^2b^2)\cos(x)\sin(x)+(a^4+a^2b^2-2\,b^4)x+2\,(a^3b-ab^3)\cos(x)}{2\,(a^5-a^3b^2)},\ \frac{2\sqrt{-a^2+b^2}\,b^3\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\sin(x)+a)}{(a^2-b^2)\cos(x)}\right)-(a^4-a^2b^2)\cos(x)\sin(x)+(a^4+a^2b^2-2\,b^4)x+2\,(a^3b-ab^3)\cos(x)}{2\,(a^5-a^3b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*b^3*log(((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2
+ b^2 + 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*a
*b*sin(x) - a^2 - b^2)) - (a^4 - a^2*b^2)*cos(x)*sin(x) + (a^4 + a^2*b^2 -
2*b^4)*x + 2*(a^3*b - a*b^3)*cos(x))/(a^5 - a^3*b^2), 1/2*(2*sqrt(-a^2 + b^
2)*b^3*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) + a)/((a^2 - b^2)*cos(x))) - (a^4
 - a^2*b^2)*cos(x)*sin(x) + (a^4 + a^2*b^2 - 2*b^4)*x + 2*(a^3*b - a*b^3)*c
os(x))/(a^5 - a^3*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin^2(x)}{a+b\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*csc(x)),x)
```

```
[Out] Integral(sin(x)**2/(a + b*csc(x)), x)
```

Giac [A]
time = 0.43, size = 112, normalized size = 1.37

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}x\right)+a}{\sqrt{-a^2+b^2}}\right)\right)b^3}{\sqrt{-a^2+b^2}\,a^3}+\frac{(a^2+2\,b^2)x}{2\,a^3}+\frac{a\tan\left(\frac{1}{2}x\right)^3+2\,b\tan\left(\frac{1}{2}x\right)^2-a\tan\left(\frac{1}{2}x\right)+2\,b}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 +
 b^2)))*b^3/(sqrt(-a^2 + b^2)*a^3) + 1/2*(a^2 + 2*b^2)*x/a^3 + (a*tan(1/2*x
)^3 + 2*b*tan(1/2*x)^2 - a*tan(1/2*x) + 2*b)/((tan(1/2*x)^2 + 1)^2*a^2)
```

Mupad [B]
time = 0.84, size = 1147, normalized size = 13.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b/sin(x)),x)
```

```
[Out] ((2*b)/a^2 - tan(x/2)/a + tan(x/2)^3/a + (2*b*tan(x/2)^2)/a^2)/(2*tan(x/2)^
2 + tan(x/2)^4 + 1) - (atan((40*b^3*tan(x/2))/(8*a^2*b + 40*b^3 + (48*b^5)/
a^2) + (48*b^5*tan(x/2))/(8*a^4*b + 48*b^5 + 40*a^2*b^3) + (8*a*b*tan(x/2))
/(8*a*b + (40*b^3)/a + (48*b^5)/a^3))*(a^2*1i + b^2*2i)*1i)/a^3 + (b^3*atan
(((b^3*(a^2 - b^2)^(1/2)*((8*(4*a^2*b^6 + 4*a^4*b^4 + a^6*b^2))/a^5 + (8*ta
n(x/2)*(2*a^8*b - 8*a^2*b^7 + 4*a^4*b^5 + 7*a^6*b^3))/a^6 + (b^3*(a^2 - b^2
)^(1/2)*(64*b^4*tan(x/2) + (8*(2*a^8*b + 2*a^6*b^3))/a^5 + (b^3*(a^2 - b^2)
^(1/2)*(32*a^3*b^2 + (8*tan(x/2)*(12*a^10*b - 8*a^8*b^3))/a^6))/(a^5 - a^3*
b^2)))/(a^5 - a^3*b^2))*1i)/(a^5 - a^3*b^2) + (b^3*(a^2 - b^2)^(1/2)*((8*(4
*a^2*b^6 + 4*a^4*b^4 + a^6*b^2))/a^5 + (8*tan(x/2)*(2*a^8*b - 8*a^2*b^7 + 4
*a^4*b^5 + 7*a^6*b^3))/a^6 - (b^3*(a^2 - b^2)^(1/2)*(64*b^4*tan(x/2) + (8*(
2*a^8*b + 2*a^6*b^3))/a^5 - (b^3*(a^2 - b^2)^(1/2)*(32*a^3*b^2 + (8*tan(x/2
)*(12*a^10*b - 8*a^8*b^3))/a^6))/(a^5 - a^3*b^2)))/(a^5 - a^3*b^2))*1i)/(a^
5 - a^3*b^2))/((16*(2*b^7 + a^2*b^5))/a^5 + (16*tan(x/2)*(8*b^8 + 8*a^2*b^6
 + 2*a^4*b^4))/a^6 + (b^3*(a^2 - b^2)^(1/2)*((8*(4*a^2*b^6 + 4*a^4*b^4 + a^
6*b^2))/a^5 + (8*tan(x/2)*(2*a^8*b - 8*a^2*b^7 + 4*a^4*b^5 + 7*a^6*b^3))/a^
```

```
6 + (b^3*(a^2 - b^2)^(1/2)*(64*b^4*tan(x/2) + (8*(2*a^8*b + 2*a^6*b^3))/a^5
 + (b^3*(a^2 - b^2)^(1/2)*(32*a^3*b^2 + (8*tan(x/2)*(12*a^10*b - 8*a^8*b^3)
)/a^6))/(a^5 - a^3*b^2)))/(a^5 - a^3*b^2)))/(a^5 - a^3*b^2) - (b^3*(a^2 - b
^2)^(1/2)*((8*(4*a^2*b^6 + 4*a^4*b^4 + a^6*b^2))/a^5 + (8*tan(x/2)*(2*a^8*b
 - 8*a^2*b^7 + 4*a^4*b^5 + 7*a^6*b^3))/a^6 - (b^3*(a^2 - b^2)^(1/2)*(64*b^4
*tan(x/2) + (8*(2*a^8*b + 2*a^6*b^3))/a^5 - (b^3*(a^2 - b^2)^(1/2)*(32*a^3*
b^2 + (8*tan(x/2)*(12*a^10*b - 8*a^8*b^3))/a^6))/(a^5 - a^3*b^2)))/(a^5 - a
^3*b^2)))/(a^5 - a^3*b^2)))*(a^2 - b^2)^(1/2)*2i)/(a^5 - a^3*b^2)
```

3.47 $\int \frac{\sin^3(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=110

$$-\frac{b(a^2+2b^2)\,x}{2a^4}-\frac{2b^4\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}}-\frac{(2a^2+3b^2)\cos(x)}{3a^3}+\frac{b\cos(x)\sin(x)}{2a^2}-\frac{\cos(x)\sin^2(x)}{3a}$$

```
[Out] -1/2*b*(a^2+2*b^2)*x/a^4-1/3*(2*a^2+3*b^2)*cos(x)/a^3+1/2*b*cos(x)*sin(x)/a
^2-1/3*cos(x)*sin(x)^2/a-2*b^4*arctanh((a+b*tan(1/2*x))/(a^2-b^2)^(1/2))/a^
4/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$,
Rules used = {3938, 4189, 4004, 3916, 2739, 632, 212}

$$\frac{b\sin(x)\cos(x)}{2a^2}-\frac{bx(a^2+2b^2)}{2a^4}-\frac{2b^4\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}}-\frac{(2a^2+3b^2)\cos(x)}{3a^3}-\frac{\sin^2(x)\cos(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Csc[x]),x]
```

```
[Out] -1/2*(b*(a^2 + 2*b^2)*x)/a^4 - (2*b^4*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b
^2]])/(a^4*Sqrt[a^2 - b^2]) - ((2*a^2 + 3*b^2)*Cos[x])/(3*a^3) + (b*Cos[x]*
Sin[x])/(2*a^2) - (Cos[x]*Sin[x]^2)/(3*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(x)}{a+b\csc(x)}\,dx &= -\frac{\cos(x)\sin^2(x)}{3a} + \frac{\int \frac{(-3b+2a\csc(x)+2b\csc^2(x))\sin^2(x)}{a+b\csc(x)}\,dx}{3a} \\
&= \frac{b\cos(x)\sin(x)}{2a^2} - \frac{\cos(x)\sin^2(x)}{3a} - \frac{\int \frac{(-2(2a^2+3b^2)-ab\csc(x)+3b^2\csc^2(x))\sin(x)}{a+b\csc(x)}\,dx}{6a^2} \\
&= -\frac{(2a^2+3b^2)\cos(x)}{3a^3} + \frac{b\cos(x)\sin(x)}{2a^2} - \frac{\cos(x)\sin^2(x)}{3a} + \frac{\int \frac{-3b(a^2+2b^2)-3ab^2\csc(x)}{a+b\csc(x)}\,dx}{6a^3} \\
&= -\frac{b(a^2+2b^2)\,x}{2a^4} - \frac{(2a^2+3b^2)\cos(x)}{3a^3} + \frac{b\cos(x)\sin(x)}{2a^2} - \frac{\cos(x)\sin^2(x)}{3a} + \frac{b^4\int \frac{\csc(x)}{a+b\csc(x)}\,\cdots}{a^4} \\
&= -\frac{b(a^2+2b^2)\,x}{2a^4} - \frac{(2a^2+3b^2)\cos(x)}{3a^3} + \frac{b\cos(x)\sin(x)}{2a^2} - \frac{\cos(x)\sin^2(x)}{3a} + \frac{b^3\int \frac{1}{1+\frac{a\sin(x)}{b}}\,\cdots}{a^4} \\
&= -\frac{b(a^2+2b^2)\,x}{2a^4} - \frac{(2a^2+3b^2)\cos(x)}{3a^3} + \frac{b\cos(x)\sin(x)}{2a^2} - \frac{\cos(x)\sin^2(x)}{3a} + \frac{(2b^3)\,\text{Subst}\Big(\cdots}{\cdots} \\
&= -\frac{b(a^2+2b^2)\,x}{2a^4} - \frac{(2a^2+3b^2)\cos(x)}{3a^3} + \frac{b\cos(x)\sin(x)}{2a^2} - \frac{\cos(x)\sin^2(x)}{3a} - \frac{(4b^3)\,\text{Subst}\Big(\cdots}{\cdots} \\
&= -\frac{b(a^2+2b^2)\,x}{2a^4} - \frac{2b^4\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}} - \frac{(2a^2+3b^2)\cos(x)}{3a^3} + \frac{b\cos(x)\sin(x)}{2a^2} - \cdots
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 98, normalized size = 0.89

$$
\frac{-6b(a^2+2b^2)\,x + \frac{24b^4\,\text{ArcTan}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 3a(3a^2+4b^2)\cos(x) + a^3\cos(3x) + 3a^2b\sin(2x)}{12a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Csc[x]),x]
```

```
[Out] (-6*b*(a^2 + 2*b^2)*x + (24*b^4*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/
Sqrt[-a^2 + b^2] - 3*a*(3*a^2 + 4*b^2)*Cos[x] + a^3*Cos[3*x] + 3*a^2*b*Sin[
2*x])/(12*a^4)
```

Maple [A]
time = 0.12, size = 145, normalized size = 1.32

method	result

default	$\frac{\frac{2\left(-\frac{a^2 b\left(\tan^5\left(\frac{x}{2}\right)\right)}{2}-a\,b^2\left(\tan^4\left(\frac{x}{2}\right)\right)+\left(-2a^3-2a\,b^2\right)\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{a^2 b\tan\left(\frac{x}{2}\right)}{2}-\frac{2a^3}{3}-a\,b^2\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}-b\left(a^2+2b^2\right)\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^4}+\frac{2b^4\arctan\left(\right.}{a^4\sqrt{-}}$
risch	$-\frac{xb}{2a^2}-\frac{x\,b^3}{a^4}-\frac{3\,\mathrm{e}^{ix}}{8a}-\frac{\mathrm{e}^{ix}b^2}{2a^3}-\frac{3\,\mathrm{e}^{-ix}}{8a}-\frac{\mathrm{e}^{-ix}b^2}{2a^3}-\frac{i b^4\ln\left(\mathrm{e}^{ix}+\frac{i\left(\sqrt{-a^2+b^2}\,b+a^2-b^2\right)}{a\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^4}+\frac{i b^4\ln\left(\mathrm{e}^{ix}+\frac{i\left(\sqrt{-a\right.}}{a\sqrt{}}\right)}{\sqrt{-a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^4*((-1/2*a^2*b*tan(1/2*x)^5-a*b^2*tan(1/2*x)^4+(-2*a^3-2*a*b^2)*tan(1/2
*x)^2+1/2*a^2*b*tan(1/2*x)-2/3*a^3-a*b^2)/(tan(1/2*x)^2+1)^3-1/2*b*(a^2+2*b
^2)*arctan(tan(1/2*x)))+2*b^4/a^4/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*
x)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 4.38, size = 329, normalized size = 2.99

$$\left[\frac{3\sqrt{a^2-b^2}\,b^4\log\left(-\frac{(a^2-2b^2)\cos(x)^2+2ab\sin(x)+a^2+b^2-2\left(b\cos(x)\sin(x)+a\cos(x)\right)\sqrt{a^2-b^2}}{a^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)+2\left(a^5-a^3b^2\right)\cos(x)^3+3\left(a^4b-a^2b^3\right)\cos(x)\sin(x)-3\left(a^4b+a^2b^3-2b^5\right)x-6\left(a^5-ab^4\right)\cos(x)}{6\left(a^6-a^4b^2\right)},\ -\frac{6\sqrt{-a^2+b^2}\,b^4\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\sin(x)+a)}{(a^2-b^2)\cos(x)}\right)-2\left(a^5-a^3b^2\right)\cos(x)^3-3\left(a^4b-a^2b^3\right)\cos(x)\sin(x)+3\left(a^4b+a^2b^3-2b^5\right)x+6\left(a^5-ab^4\right)\cos(x)}{6\left(a^6-a^4b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 - b^2)*b^4*log(-((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a
^2 + b^2 - 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 -
2*a*b*sin(x) - a^2 - b^2)) + 2*(a^5 - a^3*b^2)*cos(x)^3 + 3*(a^4*b - a^2*b^
3)*cos(x)*sin(x) - 3*(a^4*b + a^2*b^3 - 2*b^5)*x - 6*(a^5 - a*b^4)*cos(x))/
(a^6 - a^4*b^2), -1/6*(6*sqrt(-a^2 + b^2)*b^4*arctan(-sqrt(-a^2 + b^2)*(b*s
in(x) + a)/((a^2 - b^2)*cos(x))) - 2*(a^5 - a^3*b^2)*cos(x)^3 - 3*(a^4*b -
```

```
a^2*b^3)*cos(x)*sin(x) + 3*(a^4*b + a^2*b^3 - 2*b^5)*x + 6*(a^5 - a*b^4)*co
s(x))/(a^6 - a^4*b^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*csc(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 149, normalized size = 1.35

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}x\right)+a}{\sqrt{-a^2+b^2}}\right)\right)b^4}{\sqrt{-a^2+b^2}\,a^4}-\frac{(a^2b+2b^3)x}{2a^4}-\frac{3ab\tan\left(\frac{1}{2}x\right)^5+6b^2\tan\left(\frac{1}{2}x\right)^4+12a^2\tan\left(\frac{1}{2}x\right)^2+12b^2\tan\left(\frac{1}{2}x\right)^2-3ab\tan\left(\frac{1}{2}x\right)+4a^2+6b^2}{3\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 +
b^2)))*b^4/(sqrt(-a^2 + b^2)*a^4) - 1/2*(a^2*b + 2*b^3)*x/a^4 - 1/3*(3*a*b*
tan(1/2*x)^5 + 6*b^2*tan(1/2*x)^4 + 12*a^2*tan(1/2*x)^2 + 12*b^2*tan(1/2*x)
^2 - 3*a*b*tan(1/2*x) + 4*a^2 + 6*b^2)/((tan(1/2*x)^2 + 1)^3*a^3)
```

Mupad [B]
time = 0.95, size = 1218, normalized size = 11.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + b/sin(x)),x)
```

```
[Out] - ((2*(2*a^2 + 3*b^2))/(3*a^3) + (b*tan(x/2)^5)/a^2 + (2*b^2*tan(x/2)^4)/a^
3 + (4*tan(x/2)^2*(a^2 + b^2))/a^3 - (b*tan(x/2))/a^2)/(3*tan(x/2)^2 + 3*ta
n(x/2)^4 + tan(x/2)^6 + 1) - (b^4*atan(((b^4*(a^2 - b^2)^(1/2)*((8*(4*a^3*b
^8 + 4*a^5*b^6 + a^7*b^4))/a^8 + (8*tan(x/2)*(4*a^5*b^7 - 8*a^3*b^9 + 7*a^7
*b^5 + 2*a^9*b^3))/a^9 + (b^4*(a^2 - b^2)^(1/2)*((8*(2*a^8*b^4 + 2*a^10*b^2
))/a^8 + (64*b^5*tan(x/2))/a + (b^4*(a^2 - b^2)^(1/2)*(32*a^3*b^2 + (8*tan(
x/2)*(12*a^13*b - 8*a^11*b^3))/a^9))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2))*1i)
/(a^6 - a^4*b^2) + (b^4*(a^2 - b^2)^(1/2)*((8*(4*a^3*b^8 + 4*a^5*b^6 + a^7*
b^4))/a^8 + (8*tan(x/2)*(4*a^5*b^7 - 8*a^3*b^9 + 7*a^7*b^5 + 2*a^9*b^3))/a^
```

```
9 - (b^4*(a^2 - b^2)^(1/2)*((8*(2*a^8*b^4 + 2*a^10*b^2))/a^8 + (64*b^5*tan(
x/2))/a - (b^4*(a^2 - b^2)^(1/2)*(32*a^3*b^2 + (8*tan(x/2)*(12*a^13*b - 8*a
^11*b^3))/a^9))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2))/((1
6*(2*b^10 + a^2*b^8))/a^8 + (16*tan(x/2)*(8*b^11 + 8*a^2*b^9 + 2*a^4*b^7))/
a^9 + (b^4*(a^2 - b^2)^(1/2)*((8*(4*a^3*b^8 + 4*a^5*b^6 + a^7*b^4))/a^8 + (
8*tan(x/2)*(4*a^5*b^7 - 8*a^3*b^9 + 7*a^7*b^5 + 2*a^9*b^3))/a^9 + (b^4*(a^2
 - b^2)^(1/2)*((8*(2*a^8*b^4 + 2*a^10*b^2))/a^8 + (64*b^5*tan(x/2))/a + (b^
4*(a^2 - b^2)^(1/2)*(32*a^3*b^2 + (8*tan(x/2)*(12*a^13*b - 8*a^11*b^3))/a^9
))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2) - (b^4*(a^2 - b^2)^(
1/2)*((8*(4*a^3*b^8 + 4*a^5*b^6 + a^7*b^4))/a^8 + (8*tan(x/2)*(4*a^5*b^7 -
8*a^3*b^9 + 7*a^7*b^5 + 2*a^9*b^3))/a^9 - (b^4*(a^2 - b^2)^(1/2)*((8*(2*a^8
*b^4 + 2*a^10*b^2))/a^8 + (64*b^5*tan(x/2))/a - (b^4*(a^2 - b^2)^(1/2)*(32*
a^3*b^2 + (8*tan(x/2)*(12*a^13*b - 8*a^11*b^3))/a^9))/(a^6 - a^4*b^2)))/(a^
6 - a^4*b^2)))/(a^6 - a^4*b^2)))*(a^2 - b^2)^(1/2)*2i)/(a^6 - a^4*b^2) - (b
*atan((8*b^4*tan(x/2))/(8*b^4 + (40*b^6)/a^2 + (48*b^8)/a^4) + (40*b^6*tan(
x/2))/(40*b^6 + 8*a^2*b^4 + (48*b^8)/a^2) + (48*b^8*tan(x/2))/(48*b^8 + 40*
a^2*b^6 + 8*a^4*b^4))*(a^2 + 2*b^2))/a^4
```

3.48 $\int \frac{\sin^4(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=144

$$\frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5}+\frac{2b^5\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}}+\frac{b(2a^2+3b^2)\cos(x)}{3a^4}-\frac{(3a^2+4b^2)\cos(x)\sin(x)}{8a^3}+\frac{b\cos(x)\,\mathrm{si}}{3a^2}$$

```
[Out] 1/8*(3*a^4+4*a^2*b^2+8*b^4)*x/a^5+1/3*b*(2*a^2+3*b^2)*cos(x)/a^4-1/8*(3*a^2
+4*b^2)*cos(x)*sin(x)/a^3+1/3*b*cos(x)*sin(x)^2/a^2-1/4*cos(x)*sin(x)^3/a+2
*b^5*arctanh((a+b*tan(1/2*x))/(a^2-b^2)^(1/2))/a^5/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3938, 4189, 4004, 3916, 2739, 632, 212}

$$\frac{b\sin^2(x)\cos(x)}{3a^2}+\frac{2b^5\tanh^{-1}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}}+\frac{b(2a^2+3b^2)\cos(x)}{3a^4}-\frac{(3a^2+4b^2)\sin(x)\cos(x)}{8a^3}+\frac{x(3a^4+4a^2b^2+8b^4)}{8a^5}-\frac{\sin^3(x)\cos(x)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Csc[x]),x]
```

```
[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) + (2*b^5*ArcTanh[(a + b*Tan[x/2])/S
qrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]) + (b*(2*a^2 + 3*b^2)*Cos[x])/(3*a^4)
 - ((3*a^2 + 4*b^2)*Cos[x]*Sin[x])/(8*a^3) + (b*Cos[x]*Sin[x]^2)/(3*a^2) -
(Cos[x]*Sin[x]^3)/(4*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sin^4(x)}{a+b\csc(x)}\,dx = -\frac{\cos(x)\sin^3(x)}{4a} + \frac{\int \frac{(-4b+3a\csc(x)+3b\csc^2(x))\sin^3(x)}{a+b\csc(x)}\,dx}{4a}$$

$$= \frac{b\cos(x)\sin^2(x)}{3a^2} - \frac{\cos(x)\sin^3(x)}{4a} - \frac{\int \frac{(-3(3a^2+4b^2)-ab\csc(x)+8b^2\csc^2(x))\sin^2(x)}{a+b\csc(x)}\,dx}{12a^2}$$

$$= -\frac{(3a^2+4b^2)\cos(x)\sin(x)}{8a^3} + \frac{b\cos(x)\sin^2(x)}{3a^2} - \frac{\cos(x)\sin^3(x)}{4a} + \frac{\int \frac{(-8b(2a^2+3b^2)+a(9a^2-4b^2)\ldots}{}}{}$$

$$= \frac{b(2a^2+3b^2)\cos(x)}{3a^4} - \frac{(3a^2+4b^2)\cos(x)\sin(x)}{8a^3} + \frac{b\cos(x)\sin^2(x)}{3a^2} - \frac{\cos(x)\sin^3(x)}{4a} - \int$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} + \frac{b(2a^2+3b^2)\cos(x)}{3a^4} - \frac{(3a^2+4b^2)\cos(x)\sin(x)}{8a^3} + \frac{b\cos(x)\sin^2(}{3a^2}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} + \frac{b(2a^2+3b^2)\cos(x)}{3a^4} - \frac{(3a^2+4b^2)\cos(x)\sin(x)}{8a^3} + \frac{b\cos(x)\sin^2(}{3a^2}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} + \frac{b(2a^2+3b^2)\cos(x)}{3a^4} - \frac{(3a^2+4b^2)\cos(x)\sin(x)}{8a^3} + \frac{b\cos(x)\sin^2(}{3a^2}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} + \frac{b(2a^2+3b^2)\cos(x)}{3a^4} - \frac{(3a^2+4b^2)\cos(x)\sin(x)}{8a^3} + \frac{b\cos(x)\sin^2(}{3a^2}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} + \frac{2b^5\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}} + \frac{b(2a^2+3b^2)\cos(x)}{3a^4} - \frac{(3a^2+4b^2)}{8}$$

Mathematica [A]
time = 0.33, size = 129, normalized size = 0.90

$$\frac{36a^4x+48a^2b^2x+96b^4x-\frac{192b^5\operatorname{ArcTan}\left(\frac{a+b\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+24ab(3a^2+4b^2)\cos(x)-8a^3b\cos(3x)-24a^4\sin(2x)-24a^2b^2\sin(2x)+3a^4\sin(4x)}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Csc[x]),x]
```

```
[Out] (36*a^4*x + 48*a^2*b^2*x + 96*b^4*x - (192*b^5*ArcTan[(a + b*Tan[x/2])/Sqrt
[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 24*a*b*(3*a^2 + 4*b^2)*Cos[x] - 8*a^3*b*C
os[3*x] - 24*a^4*Sin[2*x] - 24*a^2*b^2*Sin[2*x] + 3*a^4*Sin[4*x])/(96*a^5)
```

Maple [A]
time = 0.15, size = 234, normalized size = 1.62

method	result
default	$-\frac{2b^5 \arctan\left(\frac{2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a^5\sqrt{-a^2+b^2}}+\frac{2\left(\left(\frac{3}{8}a^4+\frac{1}{2}a^2b^2\right)\left(\tan^7\left(\frac{x}{2}\right)\right)+a\,b^3\left(\tan^6\left(\frac{x}{2}\right)\right)+\left(\frac{1}{2}a^2b^2+\frac{11}{8}a^4\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(2a^3b+3a\,b^3\right)\left(\tan^4\left(\frac{x}{2}\right)\right.\right.}{\left(\tan^2\left(\right.\right.}$
risch	$\frac{3x}{8a}+\frac{x\,b^2}{2a^3}+\frac{x\,b^4}{a^5}+\frac{3b\,\mathrm{e}^{ix}}{8a^2}+\frac{b^3\mathrm{e}^{ix}}{2a^4}+\frac{3b\,\mathrm{e}^{-ix}}{8a^2}+\frac{b^3\mathrm{e}^{-ix}}{2a^4}+\frac{b^5\ln\left(\mathrm{e}^{ix}+\frac{ib\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,a^5}-\frac{b^5\ln\left(\mathrm{e}^{ix}+\frac{ib\sqrt{a^2-\right.}{\sqrt{a^2\right.}}{\sqrt{a^2-b^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^5/a^5/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*x)+2*a)/(-a^2+b^2)^(1/2
))+2/a^5*(((3/8*a^4+1/2*a^2*b^2)*tan(1/2*x)^7+a*b^3*tan(1/2*x)^6+(1/2*a^2*b
^2+11/8*a^4)*tan(1/2*x)^5+(2*a^3*b+3*a*b^3)*tan(1/2*x)^4+(-1/2*a^2*b^2-11/8
*a^4)*tan(1/2*x)^3+(3*a*b^3+8/3*a^3*b)*tan(1/2*x)^2+(-3/8*a^4-1/2*a^2*b^2)*
tan(1/2*x)+2/3*a^3*b+a*b^3)/(tan(1/2*x)^2+1)^4+1/8*(3*a^4+4*a^2*b^2+8*b^4)*
arctan(tan(1/2*x)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 3.39, size = 410, normalized size = 2.85

$$\left[\frac{12\sqrt{a^2-b^2}\,b^5\log\left(\frac{(a^2-2b^2)\cos(x)^2+2ab\sin(x)+a^2+b^2+2\,(b\cos(x)\sin(x)+a\cos(x))\sqrt{a^2-b^2}}{a^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)-8\,(a^5b-a^3b^3)\cos(x)^3+3\,(3a^6+a^4b^2+4a^2b^4-8b^6)x+24\,(a^5b-ab^5)\cos(x)+3\left(2\,(a^6-a^4b^2)\cos(x)^3-(5a^6-a^4b^2-4a^2b^4)\cos(x)\right)\sin(x)}{24\,(a^7-a^5b^2)},\ \frac{24\sqrt{-a^2+b^2}\,b^5\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\sin(x)+a)}{(a^2-b^2)\cos(x)}\right)-8\,(a^5b-a^3b^3)\cos(x)^3+3\,(3a^6+a^4b^2+4a^2b^4-8b^6)x+24\,(a^5b-ab^5)\cos(x)+3\left(2\,(a^6-a^4b^2)\cos(x)^3-(5a^6-a^4b^2-4a^2b^4)\cos(x)\right)\sin(x)}{24\,(a^7-a^5b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/24*(12*sqrt(a^2 - b^2)*b^5*log(((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) +
a^2 + b^2 + 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 -
 2*a*b*sin(x) - a^2 - b^2)) - 8*(a^5*b - a^3*b^3)*cos(x)^3 + 3*(3*a^6 + a^4
*b^2 + 4*a^2*b^4 - 8*b^6)*x + 24*(a^5*b - a*b^5)*cos(x) + 3*(2*(a^6 - a^4*b
^2)*cos(x)^3 - (5*a^6 - a^4*b^2 - 4*a^2*b^4)*cos(x))*sin(x))/(a^7 - a^5*b^2
```

```
), 1/24*(24*sqrt(-a^2 + b^2)*b^5*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) + a)/((
a^2 - b^2)*cos(x))) - 8*(a^5*b - a^3*b^3)*cos(x)^3 + 3*(3*a^6 + a^4*b^2 + 4
*a^2*b^4 - 8*b^6)*x + 24*(a^5*b - a*b^5)*cos(x) + 3*(2*(a^6 - a^4*b^2)*cos(
x)^3 - (5*a^6 - a^4*b^2 - 4*a^2*b^4)*cos(x))*sin(x))/(a^7 - a^5*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(x)}{a + b\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*csc(x)),x)
```

```
[Out] Integral(sin(x)**4/(a + b*csc(x)), x)
```

Giac [A]
time = 0.41, size = 252, normalized size = 1.75

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}x\right)+a}{\sqrt{-a^2+b^2}}\right)\right)b^5}{\sqrt{-a^2+b^2}\,a^5}+\frac{(3a^4+4a^2b^2+8b^4)x}{8a^5}+\frac{9a^3\tan\left(\frac{1}{2}x\right)^7+12ab^2\tan\left(\frac{1}{2}x\right)^7+24b^3\tan\left(\frac{1}{2}x\right)^6+33a^3\tan\left(\frac{1}{2}x\right)^5+12ab^2\tan\left(\frac{1}{2}x\right)^5+48a^2b\tan\left(\frac{1}{2}x\right)^4+72b^3\tan\left(\frac{1}{2}x\right)^4-33a^3\tan\left(\frac{1}{2}x\right)^3-12ab^2\tan\left(\frac{1}{2}x\right)^3+64a^2b\tan\left(\frac{1}{2}x\right)^2+72b^3\tan\left(\frac{1}{2}x\right)^2-9a^3\tan\left(\frac{1}{2}x\right)-12ab^2\tan\left(\frac{1}{2}x\right)+16a^2b+24b^3}{12\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*csc(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 +
 b^2)))*b^5/(sqrt(-a^2 + b^2)*a^5) + 1/8*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x/a^5
+ 1/12*(9*a^3*tan(1/2*x)^7 + 12*a*b^2*tan(1/2*x)^7 + 24*b^3*tan(1/2*x)^6 +
33*a^3*tan(1/2*x)^5 + 12*a*b^2*tan(1/2*x)^5 + 48*a^2*b*tan(1/2*x)^4 + 72*b^
3*tan(1/2*x)^4 - 33*a^3*tan(1/2*x)^3 - 12*a*b^2*tan(1/2*x)^3 + 64*a^2*b*tan
(1/2*x)^2 + 72*b^3*tan(1/2*x)^2 - 9*a^3*tan(1/2*x) - 12*a*b^2*tan(1/2*x) +
16*a^2*b + 24*b^3)/((tan(1/2*x)^2 + 1)^4*a^4)
```

Mupad [B]
time = 1.26, size = 1639, normalized size = 11.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + b/sin(x)),x)
```

```
[Out] ((2*(2*a^2*b + 3*b^3))/(3*a^4) - (tan(x/2)*(3*a^2 + 4*b^2))/(4*a^3) + (tan(
x/2)^7*(3*a^2 + 4*b^2))/(4*a^3) - (tan(x/2)^3*(11*a^2 + 4*b^2))/(4*a^3) + (
tan(x/2)^5*(11*a^2 + 4*b^2))/(4*a^3) + (2*b^3*tan(x/2)^6)/a^4 + (2*tan(x/2)
^4*(2*a^2*b + 3*b^3))/a^4 + (2*tan(x/2)^2*(8*a^2*b + 9*b^3))/(3*a^4))/(4*ta
n(x/2)^2 + 6*tan(x/2)^4 + 4*tan(x/2)^6 + tan(x/2)^8 + 1) - (atan((81*b^3*ta
```

```
n(x/2))/(8*((27*a^2*b)/8 + (81*b^3)/8 + (63*b^5)/(2*a^2) + (35*b^7)/a^4 + (
40*b^9)/a^6)) + (63*b^5*tan(x/2))/(2*((27*a^4*b)/8 + (63*b^5)/2 + (81*a^2*b
^3)/8 + (35*b^7)/a^2 + (40*b^9)/a^4)) + (35*b^7*tan(x/2))/((27*a^6*b)/8 + 3
5*b^7 + (63*a^2*b^5)/2 + (81*a^4*b^3)/8 + (40*b^9)/a^2) + (40*b^9*tan(x/2))
/((27*a^8*b)/8 + 40*b^9 + 35*a^2*b^7 + (63*a^4*b^5)/2 + (81*a^6*b^3)/8) + (
27*a*b*tan(x/2))/(8*((27*a*b)/8 + (81*b^3)/(8*a) + (63*b^5)/(2*a^3) + (35*b
^7)/a^5 + (40*b^9)/a^7)))*(a^4*3i + b^4*8i + a^2*b^2*4i)*1i)/(4*a^5) + (b^5
*atan(((b^5*(a^2 - b^2)^(1/2)*((32*a^4*b^10 + 32*a^6*b^8 + 32*a^8*b^6 + 12*
a^10*b^4 + (9*a^12*b^2)/2)/a^11 + (tan(x/2)*(18*a^14*b - 128*a^4*b^11 + 64*
a^6*b^9 + 64*a^8*b^7 + 104*a^10*b^5 + 39*a^12*b^3))/(2*a^12) + (b^5*(a^2 -
b^2)^(1/2)*((12*a^14*b + 16*a^10*b^5 + 4*a^12*b^3)/a^11 + (64*b^6*tan(x/2))
/a^2 + (b^5*(a^2 - b^2)^(1/2)*(32*a^3*b^2 + (tan(x/2)*(192*a^16*b - 128*a^1
4*b^3))/(2*a^12)))/(a^7 - a^5*b^2)))/(a^7 - a^5*b^2))*1i)/(a^7 - a^5*b^2) +
 (b^5*(a^2 - b^2)^(1/2)*((32*a^4*b^10 + 32*a^6*b^8 + 32*a^8*b^6 + 12*a^10*b
^4 + (9*a^12*b^2)/2)/a^11 + (tan(x/2)*(18*a^14*b - 128*a^4*b^11 + 64*a^6*b^
9 + 64*a^8*b^7 + 104*a^10*b^5 + 39*a^12*b^3))/(2*a^12) - (b^5*(a^2 - b^2)^(
1/2)*((12*a^14*b + 16*a^10*b^5 + 4*a^12*b^3)/a^11 + (64*b^6*tan(x/2))/a^2 -
 (b^5*(a^2 - b^2)^(1/2)*(32*a^3*b^2 + (tan(x/2)*(192*a^16*b - 128*a^14*b^3)
)/(2*a^12)))/(a^7 - a^5*b^2)))/(a^7 - a^5*b^2))*1i)/(a^7 - a^5*b^2))/((32*b
^13 + 40*a^2*b^11 + 24*a^4*b^9 + 9*a^6*b^7)/a^11 + (tan(x/2)*(128*b^14 + 12
8*a^2*b^12 + 128*a^4*b^10 + 48*a^6*b^8 + 18*a^8*b^6))/a^12 + (b^5*(a^2 - b^
2)^(1/2)*((32*a^4*b^10 + 32*a^6*b^8 + 32*a^8*b^6 + 12*a^10*b^4 + (9*a^12*b^
2)/2)/a^11 + (tan(x/2)*(18*a^14*b - 128*a^4*b^11 + 64*a^6*b^9 + 64*a^8*b^7
+ 104*a^10*b^5 + 39*a^12*b^3))/(2*a^12) + (b^5*(a^2 - b^2)^(1/2)*((12*a^14*
b + 16*a^10*b^5 + 4*a^12*b^3)/a^11 + (64*b^6*tan(x/2))/a^2 + (b^5*(a^2 - b^
2)^(1/2)*(32*a^3*b^2 + (tan(x/2)*(192*a^16*b - 128*a^14*b^3))/(2*a^12)))/(a
^7 - a^5*b^2)))/(a^7 - a^5*b^2)))/(a^7 - a^5*b^2) - (b^5*(a^2 - b^2)^(1/2)*
((32*a^4*b^10 + 32*a^6*b^8 + 32*a^8*b^6 + 12*a^10*b^4 + (9*a^12*b^2)/2)/a^1
1 + (tan(x/2)*(18*a^14*b - 128*a^4*b^11 + 64*a^6*b^9 + 64*a^8*b^7 + 104*a^1
0*b^5 + 39*a^12*b^3))/(2*a^12) - (b^5*(a^2 - b^2)^(1/2)*((12*a^14*b + 16*a^
10*b^5 + 4*a^12*b^3)/a^11 + (64*b^6*tan(x/2))/a^2 - (b^5*(a^2 - b^2)^(1/2)*
(32*a^3*b^2 + (tan(x/2)*(192*a^16*b - 128*a^14*b^3))/(2*a^12)))/(a^7 - a^5*
b^2)))/(a^7 - a^5*b^2)))/(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2)*2i)/(a^7 - a^5
*b^2)
```

3.49 $\int \frac{1}{(a+b\csc(c+dx))^2}\,dx$

Optimal. Leaf size=108

$$\frac{x}{a^2}+\frac{2b(2a^2-b^2)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}d}-\frac{b^2\cot(c+dx)}{a\left(a^2-b^2\right)d(a+b\csc(c+dx))}$$

```
[Out] x/a^2+2*b*(2*a^2-b^2)*arctanh((a+b*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2
/(a^2-b^2)^(3/2)/d-b^2*cot(d*x+c)/a/(a^2-b^2)/d/(a+b*csc(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3870, 4004, 3916, 2739, 632, 212}

$$\frac{2b(2a^2-b^2)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{3/2}}-\frac{b^2\cot(c+dx)}{ad\left(a^2-b^2\right)(a+b\csc(c+dx))}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csc[c + d*x])^(-2),x]
```

```
[Out] x/a^2 + (2*b*(2*a^2 - b^2)*ArcTanh[(a + b*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(a^2*(a^2 - b^2)^(3/2)*d) - (b^2*Cot[c + d*x])/(a*(a^2 - b^2)*d*(a + b*C
sc[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\csc(c+dx))^2}\,dx &= -\frac{b^2\cot(c+dx)}{a\left(a^2-b^2\right)d(a+b\csc(c+dx))}-\frac{\int \frac{-a^2+b^2+ab\csc(c+dx)}{a+b\csc(c+dx)}\,dx}{a\left(a^2-b^2\right)}\\
&= \frac{x}{a^2}-\frac{b^2\cot(c+dx)}{a\left(a^2-b^2\right)d(a+b\csc(c+dx))}-\frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{\csc(c+dx)}{a+b\csc(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}\\
&= \frac{x}{a^2}-\frac{b^2\cot(c+dx)}{a\left(a^2-b^2\right)d(a+b\csc(c+dx))}-\frac{\left(2a^2-b^2\right)\int \frac{1}{1+\frac{a\sin(c+dx)}{b}}\,dx}{a^2\left(a^2-b^2\right)}\\
&= \frac{x}{a^2}-\frac{b^2\cot(c+dx)}{a\left(a^2-b^2\right)d(a+b\csc(c+dx))}-\frac{\left(2\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx,x,\right.}{a^2\left(a^2-b^2\right)d}\\
&= \frac{x}{a^2}-\frac{b^2\cot(c+dx)}{a\left(a^2-b^2\right)d(a+b\csc(c+dx))}+\frac{\left(4\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx\right.}{a^2\left(a^2-b^2\right)}\\
&= \frac{x}{a^2}+\frac{2b\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}d}-\frac{b^2\cot(c+dx)}{a\left(a^2-b^2\right)d(a+b\csc(c+dx\right.}
\end{aligned}$$

Mathematica [A]

time = 0.46, size = 139, normalized size = 1.29

$$\frac{\csc(c+dx)\left(\frac{ab^2\cot(c+dx)}{(-a+b)(a+b)}+(c+dx)(a+b\csc(c+dx))-\frac{2b(-2a^2+b^2)\operatorname{ArcTan}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)(a+b\csc(c+dx))}{(-a^2+b^2)^{3/2}}\right)(b+a\sin(c+dx))}{a^2d(a+b\csc(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csc[c + d*x])^(-2),x]
```

```
[Out] (Csc[c + d*x]*((a*b^2*Cot[c + d*x])/((-a + b)*(a + b)) + (c + d*x)*(a + b*C
sc[c + d*x]) - (2*b*(-2*a^2 + b^2)*ArcTan[(a + b*Tan[(c + d*x)/2])/Sqrt[-a^
2 + b^2]]*(a + b*Csc[c + d*x]))/(-a^2 + b^2)^(3/2))*(b + a*Sin[c + d*x]))/(
a^2*d*(a + b*Csc[c + d*x])^2)
```

Maple [A]
time = 0.17, size = 168, normalized size = 1.56

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{2b\left(\frac{\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-2b^2}+\frac{ab}{2a^2-2b^2}}{\frac{b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b}{2}}+\frac{2\left(2a^2-b^2\right)\arctan\left(\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{\left(2a^2-2b^2\right)\sqrt{-a^2+b^2}}\right)}{a^2}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{2b\left(\frac{\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-2b^2}+\frac{ab}{2a^2-2b^2}}{\frac{b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b}{2}}+\frac{2\left(2a^2-b^2\right)\arctan\left(\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{\left(2a^2-2b^2\right)\sqrt{-a^2+b^2}}\right)}{a^2}}{d}$
risch	$\frac{x}{a^2}-\frac{2ib^2\left(ia+b\,e^{i(dx+c)}\right)}{a^2(-a^2+b^2)d\left(2b\,e^{i(dx+c)}-ia\,e^{2i(dx+c)}+ia\right)}+\frac{2b\ln\left(e^{i(dx+c)}+\frac{ib\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{b^3\ln\left(e^{i(dx+c)}+\frac{ib}{}\right.}{\sqrt{a^2-b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csc(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^2*arctan(tan(1/2*d*x+1/2*c))-2/a^2*b*((1/2*a^2/(a^2-b^2)*tan(1/2*d
*x+1/2*c)+1/2*a*b/(a^2-b^2))/(1/2*b*tan(1/2*d*x+1/2*c)^2+a*tan(1/2*d*x+1/2*
c)+1/2*b)+2*(2*a^2-b^2)/(2*a^2-2*b^2)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(
1/2*d*x+1/2*c)+2*a)/(-a^2+b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(103) = 206$.
time = 3.69, size = 493, normalized size = 4.56

$$\left[\frac{2\,(a^5-2\,a^3b^2+ab^4)dx\sin\left(dx+c\right)+2\,(a^4b-2\,a^2b^3+b^5)dx+\left(2\,a^2b^2-b^4+(2\,a^3b-ab^3)\sin\left(dx+c\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{(a^2-2\,b^2)\cos(dx+c)^2+2\,ab\sin(dx+c)+a^2+b^2+2\,(b\cos(dx+c)\sin(dx+c)+a\cos(dx+c))\sqrt{a^2-b^2}}{a^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)-2\,(a^3b^2-ab^4)\cos\left(dx+c\right)}{2\,\left((a^7-2\,a^5b^2+a^3b^4)d\sin\left(dx+c\right)+(a^6b-2\,a^4b^3+a^2b^5)d\right)},\ \frac{(a^5-2\,a^3b^2+ab^4)dx\sin\left(dx+c\right)+(a^4b-2\,a^2b^3+b^5)dx+\left(2\,a^2b^2-b^4+(2\,a^3b-ab^3)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\sin(dx+c)+a)}{(a^2-b^2)\cos(dx+c)}\right)-(a^3b^2-ab^4)\cos\left(dx+c\right)}{(a^7-2\,a^5b^2+a^3b^4)d\sin\left(dx+c\right)+(a^6b-2\,a^4b^3+a^2b^5)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*sin(d*x + c) + 2*(a^4*b - 2*a^2*b^3 +
 b^5)*d*x + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*sin(d*x + c))*sqrt(a^2 - b
^2)*log(((a^2 - 2*b^2)*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) + a^2 + b^2 + 2*
(b*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c))*sqrt(a^2 - b^2))/(a^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^3*b^2 - a*b^4)*cos(d*x +
 c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*sin(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2
*b^5)*d), ((a^5 - 2*a^3*b^2 + a*b^4)*d*x*sin(d*x + c) + (a^4*b - 2*a^2*b^3
+ b^5)*d*x + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*sin(d*x + c))*sqrt(-a^2 +
 b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(d*x + c) + a)/((a^2 - b^2)*cos(d*x +
c))) - (a^3*b^2 - a*b^4)*cos(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*sin(d
*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\csc\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))**2,x)
```

```
[Out] Integral((a + b*csc(c + d*x))**(-2), x)
```

Giac [A]
time = 0.44, size = 158, normalized size = 1.46

$$-\frac{\frac{2\,(2\,a^2b-b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-a^2b^2)\sqrt{-a^2+b^2}}+\frac{2\,\left(ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^2\right)}{(a^3-ab^2)\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}-\frac{dx+c}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(2*a^2*b - b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(b) + arctan((b*ta
n(1/2*d*x + 1/2*c) + a)/sqrt(-a^2 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2
)) + 2*(a*b*tan(1/2*d*x + 1/2*c) + b^2)/((a^3 - a*b^2)*(b*tan(1/2*d*x + 1/2
*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)) - (d*x + c)/a^2)/d
```

Mupad [B]
time = 4.18, size = 2677, normalized size = 24.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/sin(c + d*x))^2,x)
```

```
[Out] (b*atan(((b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^6 - 2*a^3*b
^4 + a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) - (32*tan(c/2 + (d*x)/2)*(2*a*b^
7 - 2*a^7*b - 8*a^3*b^5 + 9*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*(2*a
^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a^8*b - a^6*b^3))/(a^6 + a^2*b^
4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(2*a^4*b^6 - 6*a^6*b^4 + 4*a^8*b^2)
)/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2
)*((32*(a^5*b^6 - 2*a^7*b^4 + a^9*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*t
an(c/2 + (d*x)/2)*(3*a^11*b - 2*a^5*b^7 + 7*a^7*b^5 - 8*a^9*b^3))/(a^7 + a^
3*b^4 - 2*a^5*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b
^6 + 3*a^4*b^4 - 3*a^6*b^2))*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) -
(b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(2*a*b
^7 - 2*a^7*b - 8*a^3*b^5 + 9*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (32*(a
*b^6 - 2*a^3*b^4 + a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (b*(2*a^2 - b^2)
*((a + b)^3*(a - b)^3)^(1/2)*((32*(a^8*b - a^6*b^3))/(a^6 + a^2*b^4 - 2*a^4
*b^2) + (32*tan(c/2 + (d*x)/2)*(2*a^4*b^6 - 6*a^6*b^4 + 4*a^8*b^2))/(a^7 +
a^3*b^4 - 2*a^5*b^2) - (b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a
^5*b^6 - 2*a^7*b^4 + a^9*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 +
(d*x)/2)*(3*a^11*b - 2*a^5*b^7 + 7*a^7*b^5 - 8*a^9*b^3))/(a^7 + a^3*b^4 - 2
*a^5*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^
4*b^4 - 3*a^6*b^2))*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))/((64*(b^5
- 2*a^2*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (64*tan(c/2 + (d*x)/2)*(2*b^6 -
 6*a^2*b^4 + 4*a^4*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*(2*a^2 - b^2)*((a
 + b)^3*(a - b)^3)^(1/2)*((32*(a*b^6 - 2*a^3*b^4 + a^5*b^2))/(a^6 + a^2*b^4
 - 2*a^4*b^2) - (32*tan(c/2 + (d*x)/2)*(2*a*b^7 - 2*a^7*b - 8*a^3*b^5 + 9*a
^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*(2*a^2 - b^2)*((a + b)^3*(a - b)^
3)^(1/2)*((32*(a^8*b - a^6*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2
+ (d*x)/2)*(2*a^4*b^6 - 6*a^6*b^4 + 4*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2)
 + (b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a^5*b^6 - 2*a^7*b^4 +
 a^9*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(3*a^11*b -
 2*a^5*b^7 + 7*a^7*b^5 - 8*a^9*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2)))/(a^8 - a
^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))
```

```
/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) + (b*(2*a^2 - b^2)*((a + b)^3*(a -
 b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(2*a*b^7 - 2*a^7*b - 8*a^3*b^5 + 9*a^5
*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (32*(a*b^6 - 2*a^3*b^4 + a^5*b^2))/(a^
6 + a^2*b^4 - 2*a^4*b^2) + (b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((3
2*(a^8*b - a^6*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(
2*a^4*b^6 - 6*a^6*b^4 + 4*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (b*(2*a^2
 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a^5*b^6 - 2*a^7*b^4 + a^9*b^2))/(
a^6 + a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(3*a^11*b - 2*a^5*b^7 +
 7*a^7*b^5 - 8*a^9*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2)))/(a^8 - a^2*b^6 + 3*a
^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*
b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i
)/(d*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)) - ((2*b*tan(c/2 + (d*x)/2))/(
a^2 - b^2) + (2*b^2)/(a*(a^2 - b^2)))/(d*(b + 2*a*tan(c/2 + (d*x)/2) + b*ta
n(c/2 + (d*x)/2)^2)) - (2*atan((64*a^5*b*tan(c/2 + (d*x)/2))/((64*a^3*b^9)/
(a^6 + a^2*b^4 - 2*a^4*b^2) - (192*a^5*b^7)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (
128*a^7*b^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (64*a^9*b^3)/(a^6 + a^2*b^4 - 2*
a^4*b^2) - (64*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b^2)) - (64*a*b^5*tan(c/2 + (
d*x)/2))/((64*a^3*b^9)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (192*a^5*b^7)/(a^6 + a
^2*b^4 - 2*a^4*b^2) + (128*a^7*b^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (64*a^9*b
^3)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (64*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b^2))
+ (64*a^3*b^3*tan(c/2 + (d*x)/2))/((64*a^3*b^9)/(a^6 + a^2*b^4 - 2*a^4*b^2)
 - (192*a^5*b^7)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (128*a^7*b^5)/(a^6 + a^2*b^4
 - 2*a^4*b^2) + (64*a^9*b^3)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (64*a^11*b)/(a^6
 + a^2*b^4 - 2*a^4*b^2))))/(a^2*d)
```

3.50 $\int \frac{1}{(a+b\csc(c+dx))^3}\,dx$

Optimal. Leaf size=170

$$\frac{x}{a^3}+\frac{b(6a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}-\frac{b^2\cot(c+dx)}{2a\left(a^2-b^2\right)d(a+b\csc(c+dx))^2}-\frac{b^2(5a^2-2b^2)\,\text{co}}{2a^2\left(a^2-b^2\right)^2d(a+}$$

```
[Out] x/a^3+b*(6*a^4-5*a^2*b^2+2*b^4)*arctanh((a+b*tan(1/2*d*x+1/2*c))/(a^2-b^2)^
(1/2))/a^3/(a^2-b^2)^(5/2)/d-1/2*b^2*cot(d*x+c)/a/(a^2-b^2)/d/(a+b*csc(d*x+
c))^2-1/2*b^2*(5*a^2-2*b^2)*cot(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*csc(d*x+c))
```

Rubi [A]
time = 0.21, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3870, 4145, 4004, 3916, 2739, 632, 212}

$$\frac{x}{a^3}-\frac{b^2(5a^2-2b^2)\cot(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\csc(c+dx))}-\frac{b^2\cot(c+dx)}{2ad\left(a^2-b^2\right)(a+b\csc(c+dx))^2}+\frac{b(6a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csc[c + d*x])^(-3),x]
```

```
[Out] x/a^3 + (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(a + b*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/(a^3*(a^2 - b^2)^(5/2)*d) - (b^2*Cot[c + d*x])/(2*a*(a^2 - b
^2)*d*(a + b*Csc[c + d*x])^2) - (b^2*(5*a^2 - 2*b^2)*Cot[c + d*x])/(2*a^2*(
a^2 - b^2)^2*d*(a + b*Csc[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+b\csc(c+dx))^3}\,dx &= -\frac{b^2\cot(c+dx)}{2a\left(a^2-b^2\right)d(a+b\csc(c+dx))^2} - \frac{\int \frac{-2\left(a^2-b^2\right)+2ab\csc(c+dx)-b^2\csc^2(c+dx)}{(a+b\csc(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)} \\
&= -\frac{b^2\cot(c+dx)}{2a\left(a^2-b^2\right)d(a+b\csc(c+dx))^2} - \frac{b^2(5a^2-2b^2)\cot(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\csc(c+dx))} + \frac{\int 2(\cdots}{\cdots} \\
&= \frac{x}{a^3} - \frac{b^2\cot(c+dx)}{2a\left(a^2-b^2\right)d(a+b\csc(c+dx))^2} - \frac{b^2(5a^2-2b^2)\cot(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\csc(c+dx))} - \cdots \\
&= \frac{x}{a^3} - \frac{b^2\cot(c+dx)}{2a\left(a^2-b^2\right)d(a+b\csc(c+dx))^2} - \frac{b^2(5a^2-2b^2)\cot(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\csc(c+dx))} - \cdots \\
&= \frac{x}{a^3} - \frac{b^2\cot(c+dx)}{2a\left(a^2-b^2\right)d(a+b\csc(c+dx))^2} - \frac{b^2(5a^2-2b^2)\cot(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\csc(c+dx))} - \cdots \\
&= \frac{x}{a^3} - \frac{b^2\cot(c+dx)}{2a\left(a^2-b^2\right)d(a+b\csc(c+dx))^2} - \frac{b^2(5a^2-2b^2)\cot(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\csc(c+dx))} + \cdots \\
&= \frac{x}{a^3} + \frac{b(6a^4-5a^2b^2+2b^4)\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d} - \frac{b^2\cot(c+dx}{2a\left(a^2-b^2\right)d(a+b\csc\cdots}\end{aligned}$$

Mathematica [A]
time = 1.18, size = 216, normalized size = 1.27

$$\frac{\csc^2(c+dx)(b+a\sin(c+dx))\left(\frac{ab^3\cot(c+dx)}{(a-b)(a+b)}-\frac{3ab^2(2a^2-b^2)\cot(c+dx)(b+a\sin(c+dx))}{(a-b)^2(a+b)^2}+2(c+dx)\csc(c+dx)(b+a\sin(c+dx))^2-\frac{2b(6a^4-5a^2b^2+2b^4)\operatorname{ArcTan}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)\csc(c+dx)(b+a\sin(c+dx))^2}{(-a^2+b^2)^{5/2}}\right)}{2a^3d(a+b\csc(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csc[c + d*x])^(-3),x]
```

```
[Out] (Csc[c + d*x]^2*(b + a*Sin[c + d*x])*((a*b^3*Cot[c + d*x])/((a - b)*(a + b)
) - (3*a*b^2*(2*a^2 - b^2)*Cot[c + d*x]*(b + a*Sin[c + d*x]))/((a - b)^2*(a
 + b)^2) + 2*(c + d*x)*Csc[c + d*x]*(b + a*Sin[c + d*x])^2 - (2*b*(6*a^4 -
5*a^2*b^2 + 2*b^4)*ArcTan[(a + b*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]*Csc[c
+ d*x]*(b + a*Sin[c + d*x])^2)/(-a^2 + b^2)^(5/2)))/(2*a^3*d*(a + b*Csc[c +
 d*x])^3)
```

Maple [A]
time = 0.26, size = 314, normalized size = 1.85

method	result
derivativedivides	$-\frac{\frac{2b\left(\frac{\frac{4a^2b(4a^2-b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^4-16a^2b^2+8b^4}+\frac{4a(10a^4+a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^4-16a^2b^2+8b^4}+\frac{4a^2b(16a^2-7b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4-16a^2b^2+8b^4}+\frac{4ab^2(5a^2-2b^2)}{8a^4-16a^2b^2+8b^4}}{\left(b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+b\right)^2}}{a^3}}{d}$
default	$-\frac{\frac{2b\left(\frac{\frac{4a^2b(4a^2-b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^4-16a^2b^2+8b^4}+\frac{4a(10a^4+a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^4-16a^2b^2+8b^4}+\frac{4a^2b(16a^2-7b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4-16a^2b^2+8b^4}+\frac{4ab^2(5a^2-2b^2)}{8a^4-16a^2b^2+8b^4}}{\left(b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+b\right)^2}}{a^3}}{d}$
risch	$\frac{x}{a^3}-\frac{ib^2(7ia^3b\,e^{3i(dx+c)}-4iab^3e^{3i(dx+c)}-17ia^3b\,e^{i(dx+c)}+8ib^3a\,e^{i(dx+c)}-6a^4e^{2i(dx+c)}-9a^2b^2e^{2i(dx+c)}+6b^4e^{2i(dx}}{(2b\,e^{i(dx+c)}-ia\,e^{2i(dx+c)}+ia)^2(-a^2+b^2)^2d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csc(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^3*b*(4*(1/8*a^2*b*(4*a^2-b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/2*d*x+1/2
*c)^3+1/8*a*(10*a^4+a^2*b^2-2*b^4)/(a^4-2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^2
+1/8*a^2*b*(16*a^2-7*b^2)/(a^4-2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)+1/8*a*b^2*
(5*a^2-2*b^2)/(a^4-2*a^2*b^2+b^4))/(b*tan(1/2*d*x+1/2*c)^2+2*a*tan(1/2*d*x+
1/2*c)+b)^2+2*(6*a^4-5*a^2*b^2+2*b^4)/(4*a^4-8*a^2*b^2+4*b^4)/(-a^2+b^2)^(1
/2)*arctan(1/2*(2*b*tan(1/2*d*x+1/2*c)+2*a)/(-a^2+b^2)^(1/2)))+2/a^3*arctan
(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(161) = 322.
time = 3.33, size = 933, normalized size = 5.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2 - 4*(a^8
 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d*x - (6*a^6*b + a^4*b^3 - 3*a^2*b^5 + 2*b^
7 - (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3
*b^4 + 2*a*b^6)*sin(d*x + c))*sqrt(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(d*x +
c)^2 + 2*a*b*sin(d*x + c) + a^2 + b^2 + 2*(b*cos(d*x + c)*sin(d*x + c) + a*
cos(d*x + c))*sqrt(a^2 - b^2))/(a^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*(5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(d*x + c) - 2*(4*(a^7*b
 - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x - 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)
*cos(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*co
s(d*x + c)^2 - 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*sin(d*x + c)
- (a^11 - 2*a^9*b^2 + 2*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(a^8 - 3*a^6*b^2 + 3*
a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2 - 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^
8)*d*x - (6*a^6*b + a^4*b^3 - 3*a^2*b^5 + 2*b^7 - (6*a^6*b - 5*a^4*b^3 + 2*
a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*sin(d*x + c))
*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(d*x + c) + a)/((a^2 - b^2
)*cos(d*x + c))) + (5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(d*x + c) - (4*(a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x - 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^
6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*
cos(d*x + c)^2 - 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*sin(d*x + c
) - (a^11 - 2*a^9*b^2 + 2*a^5*b^6 - a^3*b^8)*d)]
```

<u>Sympy</u> [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\csc\left(c + dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))**3,x)
```

```
[Out] Integral((a + b*csc(c + d*x))**(-3), x)
```

<u>Giac</u> [A]
time = 0.42, size = 297, normalized size = 1.75

$$-\frac{\frac{\left(6\,a^4b-5\,a^2b^3+2\,b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{-a^2+b^2}}+\frac{4\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+10\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+16\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^2b^3-2\,b^5}{\left(a^6-2\,a^4b^2+a^2b^4\right)\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)^2}-\frac{dx+c}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(b) +
arctan((b*tan(1/2*d*x + 1/2*c) + a)/sqrt(-a^2 + b^2)))/((a^7 - 2*a^5*b^2 +
```

```
a^3*b^4)*sqrt(-a^2 + b^2)) + (4*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - a*b^4*tan(
1/2*d*x + 1/2*c)^3 + 10*a^4*b*tan(1/2*d*x + 1/2*c)^2 + a^2*b^3*tan(1/2*d*x
+ 1/2*c)^2 - 2*b^5*tan(1/2*d*x + 1/2*c)^2 + 16*a^3*b^2*tan(1/2*d*x + 1/2*c)
 - 7*a*b^4*tan(1/2*d*x + 1/2*c) + 5*a^2*b^3 - 2*b^5)/((a^6 - 2*a^4*b^2 + a^
2*b^4)*(b*tan(1/2*d*x + 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)^2) - (d*x
+ c)/a^3)/d
```

Mupad [B]
time = 8.80, size = 2500, normalized size = 14.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/sin(c + d*x))^3,x)
```

```
[Out] ((2*b^5 - 5*a^2*b^3)/(a^2*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)*(7
*b^4 - 16*a^2*b^2))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^3*(b^
4 - 4*a^2*b^2))/(a*(a^4 + b^4 - 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^2*(5*a^2*
b - 2*b^3)*(2*a^2 + b^2))/(a^2*(a^4 + b^4 - 2*a^2*b^2)))/(d*(tan(c/2 + (d*x
)/2)^2*(4*a^2 + 2*b^2) + b^2*tan(c/2 + (d*x)/2)^4 + b^2 + 4*a*b*tan(c/2 + (
d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2))) + (2*atan((((8*(4*a^2*b^10 - 16*a^4*
b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(a^13 + a^5*b^8 - 4*a^7*b^6 +
6*a^9*b^4 - 4*a^11*b^2) - (((8*(4*a^14*b + 2*a^6*b^9 - 4*a^8*b^7 + 6*a^10*b
^5 - 8*a^12*b^3))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) - (
((8*(4*a^8*b^10 - 16*a^10*b^8 + 24*a^12*b^6 - 16*a^14*b^4 + 4*a^16*b^2))/(a
^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (8*tan(c/2 + (d*x)/2)
*(12*a^18*b - 8*a^8*b^11 + 44*a^10*b^9 - 96*a^12*b^7 + 104*a^14*b^5 - 56*a^
16*b^3))/(a^14 + a^6*b^8 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))*1i)/a^3 +
(8*tan(c/2 + (d*x)/2)*(8*a^6*b^10 - 36*a^8*b^8 + 72*a^10*b^6 - 68*a^12*b^4
+ 24*a^14*b^2))/(a^14 + a^6*b^8 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))*1i)
/a^3 + (8*tan(c/2 + (d*x)/2)*(8*a^12*b - 8*a^2*b^11 + 44*a^4*b^9 - 105*a^6*
b^7 + 124*a^8*b^5 - 72*a^10*b^3))/(a^14 + a^6*b^8 - 4*a^8*b^6 + 6*a^10*b^4
- 4*a^12*b^2))/a^3 + ((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4
 + 4*a^10*b^2))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (((
((8*(4*a^8*b^10 - 16*a^10*b^8 + 24*a^12*b^6 - 16*a^14*b^4 + 4*a^16*b^2))/(a
^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (8*tan(c/2 + (d*x)/2)
*(12*a^18*b - 8*a^8*b^11 + 44*a^10*b^9 - 96*a^12*b^7 + 104*a^14*b^5 - 56*a^
16*b^3))/(a^14 + a^6*b^8 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))*1i)/a^3 +
(8*(4*a^14*b + 2*a^6*b^9 - 4*a^8*b^7 + 6*a^10*b^5 - 8*a^12*b^3))/(a^13 + a^
5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^6*
b^10 - 36*a^8*b^8 + 72*a^10*b^6 - 68*a^12*b^4 + 24*a^14*b^2))/(a^14 + a^6*b
^8 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))*1i)/a^3 + (8*tan(c/2 + (d*x)/2)*
(8*a^12*b - 8*a^2*b^11 + 44*a^4*b^9 - 105*a^6*b^7 + 124*a^8*b^5 - 72*a^10*b
^3))/(a^14 + a^6*b^8 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))/a^3)/((16*(2*b
^9 - 13*a^2*b^7 + 26*a^4*b^5 - 24*a^6*b^3))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6
```

*a^9*b^4 - 4*a^11*b^2) - (((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^
8*b^4 + 4*a^10*b^2))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2)
- (((8*(4*a^14*b + 2*a^6*b^9 - 4*a^8*b^7 + 6*a^10*b^5 - 8*a^12*b^3))/(a^13
+ a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) - (((8*(4*a^8*b^10 - 16*a^1
0*b^8 + 24*a^12*b^6 - 16*a^14*b^4 + 4*a^16*b^2))/(a^13 + a^5*b^8 - 4*a^7*b^
6 + 6*a^9*b^4 - 4*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(12*a^18*b - 8*a^8*b^11
 + 44*a^10*b^9 - 96*a^12*b^7 + 104*a^14*b^5 - 56*a^16*b^3))/(a^14 + a^6*b^8
 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))*1i)/a^3 + (8*tan(c/2 + (d*x)/2)*(8
*a^6*b^10 - 36*a^8*b^8 + 72*a^10*b^6 - 68*a^12*b^4 + 24*a^14*b^2))/(a^14 +
a^6*b^8 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))*1i)/a^3 + (8*tan(c/2 + (d*x
)/2)*(8*a^12*b - 8*a^2*b^11 + 44*a^4*b^9 - 105*a^6*b^7 + 124*a^8*b^5 - 72*a
^10*b^3))/(a^14 + a^6*b^8 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))*1i)/a^3 +
 (((8*(4*a^2*b^10 - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(a^
13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (((((8*(4*a^8*b^10 - 1
6*a^10*b^8 + 24*a^12*b^6 - 16*a^14*b^4 + 4*a^16*b^2))/(a^13 + a^5*b^8 - 4*a
^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(12*a^18*b - 8*a^8
*b^11 + 44*a^10*b^9 - 96*a^12*b^7 + 104*a^14*b^5 - 56*a^16*b^3))/(a^14 + a^
6*b^8 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))*1i)/a^3 + (8*(4*a^14*b + 2*a^
6*b^9 - 4*a^8*b^7 + 6*a^10*b^5 - 8*a^12*b^3))/(a^13 + a^5*b^8 - 4*a^7*b^6 +
 6*a^9*b^4 - 4*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^6*b^10 - 36*a^8*b^8 +
 72*a^10*b^6 - 68*a^12*b^4 + 24*a^14*b^2))/(a^14 + a^6*b^8 - 4*a^8*b^6 + 6*
a^10*b^4 - 4*a^12*b^2))*1i)/a^3 + (8*tan(c/2 + (d*x)/2)*(8*a^12*b - 8*a^2*b
^11 + 44*a^4*b^9 - 105*a^6*b^7 + 124*a^8*b^5 - 72*a^10*b^3))/(a^14 + a^6*b^
8 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))*1i)/a^3 + (16*tan(c/2 + (d*x)/2)*
(8*b^10 - 36*a^2*b^8 + 72*a^4*b^6 - 68*a^6*b^4 + 24*a^8*b^2))/(a^14 + a^6*b
^8 - 4*a^8*b^6 + 6*a^10*b^4 - 4*a^12*b^2))))/(a^3*d) + (b*atan(((b*((a + b)
^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*((8*(4*a^2*b^10 - 16*a^4*b^
8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^10*b^2))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*
a^9*b^4 - 4*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^12*b - 8*a^2*b^11 + 44*a
^4*b^9 - 105*a^6*b^7 + 124*a^8*b^5 - 72*a^10*b^3))/(a^14 + a^6*b^8 - 4*a^8*
b^6 + 6*a^10*b^4 - 4*a^12*b^2) - (b*((a + b)^5*(a - b)^5)^(1/2)*((8*(4*a^14
*b + 2*a^6*b^9 - 4*a^8*b^7 + 6*a^10*b^5 - 8*a^12*b^3))/(a^13 + a^5*b^8 - 4*
a^7*b^6 + 6*a^9*b^4 - 4*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^6*b^10 - 36*
a^8*b^8 + 72*a^10*b^6 - 68*a^12*b^4 + 24*a^14*b^2))/(a^14 + a^6*b^8 - 4*a^8
*b^6 + 6*a^10*b^4 - 4*a^12*b^2) - (b*((8*(4*a^8*b^10 - 16*a^10*b^8 + 24*a^1
2*b^6 - 16*a^14*b^4 + 4*a^16*b^2))/(a^13 + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4
- 4*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(12*a^18*...

3.51 $\int \frac{1}{(a+b\csc(c+dx))^4}\,dx$

Optimal. Leaf size=239

$$\frac{x}{a^4}+\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{7/2}d}-\frac{b^2\cot(c+dx)}{3a\left(a^2-b^2\right)d(a+b\csc(c+dx))^3}-\frac{b^2(8a^2}{6a^2\left(a^2-b\right.}$$

```
[Out] x/a^4+b*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)*arctanh((a+b*tan(1/2*d*x+1/2*c))/
(a^2-b^2)^(1/2))/a^4/(a^2-b^2)^(7/2)/d-1/3*b^2*cot(d*x+c)/a/(a^2-b^2)/d/(a+
b*csc(d*x+c))^3-1/6*b^2*(8*a^2-3*b^2)*cot(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*csc
(d*x+c))^2-1/6*b^2*(26*a^4-17*a^2*b^2+6*b^4)*cot(d*x+c)/a^3/(a^2-b^2)^3/d/(
a+b*csc(d*x+c))
```

Rubi [A]

time = 0.34, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3870, 4145, 4004, 3916, 2739, 632, 212}

$$\frac{x}{a^4}-\frac{b^2\left(8a^2-3b^2\right)\cot(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\csc(c+dx))^2}-\frac{b^2\cot(c+dx)}{3ad\left(a^2-b^2\right)(a+b\csc(c+dx))^3}+\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d\left(a^2-b^2\right)^{7/2}}-\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\cot(c+dx)}{6a^3d\left(a^2-b^2\right)^3(a+b\csc(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csc[c + d*x])^(-4),x]
```

```
[Out] x/a^4 + (b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*ArcTanh[(a + b*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2 - b^2)^(7/2)*d) - (b^2*Cot[c + d*x])/(
3*a*(a^2 - b^2)*d*(a + b*Csc[c + d*x])^3) - (b^2*(8*a^2 - 3*b^2)*Cot[c + d*
x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Csc[c + d*x])^2) - (b^2*(26*a^4 - 17*a^2*
b^2 + 6*b^4)*Cot[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Csc[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\csc(c+dx))^4}\,dx &= -\frac{b^2\cot(c+dx)}{3a\,(a^2-b^2)\,d(a+b\csc(c+dx))^3} - \frac{\int \frac{-3(a^2-b^2)+3ab\csc(c+dx)-2b^2\csc^2(c+dx)}{(a+b\csc(c+dx))^3}\,dx}{3a\,(a^2-b^2)} \\
&= -\frac{b^2\cot(c+dx)}{3a\,(a^2-b^2)\,d(a+b\csc(c+dx))^3} - \frac{b^2(8a^2-3b^2)\cot(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\csc(c+dx))^2} + \frac{\int}{} \\
&= -\frac{b^2\cot(c+dx)}{3a\,(a^2-b^2)\,d(a+b\csc(c+dx))^3} - \frac{b^2(8a^2-3b^2)\cot(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\csc(c+dx))^2} - \frac{b}{6} \\
&= \frac{x}{a^4} - \frac{b^2\cot(c+dx)}{3a\,(a^2-b^2)\,d(a+b\csc(c+dx))^3} - \frac{b^2(8a^2-3b^2)\cot(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\csc(c+dx))^2} \\
&= \frac{x}{a^4} - \frac{b^2\cot(c+dx)}{3a\,(a^2-b^2)\,d(a+b\csc(c+dx))^3} - \frac{b^2(8a^2-3b^2)\cot(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\csc(c+dx))^2} \\
&= \frac{x}{a^4} - \frac{b^2\cot(c+dx)}{3a\,(a^2-b^2)\,d(a+b\csc(c+dx))^3} - \frac{b^2(8a^2-3b^2)\cot(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\csc(c+dx))^2} \\
&= \frac{x}{a^4} - \frac{b^2\cot(c+dx)}{3a\,(a^2-b^2)\,d(a+b\csc(c+dx))^3} - \frac{b^2(8a^2-3b^2)\cot(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\csc(c+dx))^2} \\
&= \frac{x}{a^4} + \frac{b(8a^6-8a^4b^2+7a^2b^4-2b^6)\tanh^{-1}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\,(a^2-b^2)^{7/2}\,d} - \frac{b^2\,\mathrm{c}}{3a\,(a^2-b^2)\,a}
\end{aligned}$$

Mathematica [A]
time = 2.13, size = 279, normalized size = 1.17

$$\frac{\csc^3(c+dx)(b+a\sin(c+dx))\left(\frac{2ab^4\cot(c+dx)}{(-a+b)(a+b)}+\frac{ab^3(12a^2-7b^2)\cot(c+dx)(b+a\sin(c+dx))}{(a-b)^2(a+b)^2}-\frac{ab^2(36a^4-32a^2b^2+11b^4)\cot(c+dx)(b+a\sin(c+dx))^2}{(a-b)^3(a+b)^3}+6(c+dx)\csc(c+dx)(b+a\sin(c+dx))^3-\frac{6b(-8a^6+8a^4b^2-7a^2b^4+2b^6)\mathrm{ArcTan}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)\csc(c+dx)(b+a\sin(c+dx))^3}{(-a^2+b^2)^{7/2}}\right)}{6a^4d(a+b\csc(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csc[c + d*x])^(-4),x]
```

```
[Out] (Csc[c + d*x]^3*(b + a*Sin[c + d*x])*((2*a*b^4*Cot[c + d*x])/((-a + b)*(a +
 b)) + (a*b^3*(12*a^2 - 7*b^2)*Cot[c + d*x]*(b + a*Sin[c + d*x]))/((a - b)^
2*(a + b)^2) - (a*b^2*(36*a^4 - 32*a^2*b^2 + 11*b^4)*Cot[c + d*x]*(b + a*Si
n[c + d*x])^2)/((a - b)^3*(a + b)^3) + 6*(c + d*x)*Csc[c + d*x]*(b + a*Sin[
c + d*x])^3 - (6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTan[(a + b*T
an[(c + d*x)/2])/Sqrt[-a^2 + b^2]]*Csc[c + d*x]*(b + a*Sin[c + d*x])^3)/(-a
^2 + b^2)^(7/2)))/(6*a^4*d*(a + b*Csc[c + d*x])^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(228) = 456$.
time = 0.39, size = 545, normalized size = 2.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*csc(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^4*arctan(tan(1/2*d*x+1/2*c))-2/a^4*b*(8*(1/16*b^2*a^2*(6*a^4-2*a^2
*b^2+b^4)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^5+1/16*a*b*(28*a
^6-4*a^4*b^2-a^2*b^4+2*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c
)^4+1/24*a^2*(52*a^6+44*a^4*b^2-39*a^2*b^4+18*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4
-b^6)*tan(1/2*d*x+1/2*c)^3+1/8*a*b*(38*a^6-19*a^4*b^2+4*a^2*b^4+2*b^6)/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^2+1/16*(46*a^4-32*a^2*b^2+11*b
^4)*a^2*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)+1/48*a*b^3*(26
*a^4-17*a^2*b^2+6*b^4)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6))/(b*tan(1/2*d*x+1/2*c)
^2+2*a*tan(1/2*d*x+1/2*c)+b)^3+4*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)/(8*a^6-2
4*a^4*b^2+24*a^2*b^4-8*b^6)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*d*x+1/
2*c)+2*a)/(-a^2+b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(228) = 456$.
time = 4.26, size = 1554, normalized size = 6.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*cos(d*
x + c)^2 - 2*(36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 - 11*a^3*b^8)*cos(d*x +
c)^3 - 12*(3*a^10*b - 11*a^8*b^3 + 14*a^6*b^5 - 6*a^4*b^7 - a^2*b^9 + b^11)
*d*x - 3*(24*a^8*b^2 - 16*a^6*b^4 + 13*a^4*b^6 + a^2*b^8 - 2*b^10 - 3*(8*a^
8*b^2 - 8*a^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8)*cos(d*x + c)^2 + (8*a^9*b + 16*a
```

```
^7*b^3 - 17*a^5*b^5 + 19*a^3*b^7 - 6*a*b^9 - (8*a^9*b - 8*a^7*b^3 + 7*a^5*b
^5 - 2*a^3*b^7)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*log(((a^2 - 2
*b^2)*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) + a^2 + b^2 + 2*(b*cos(d*x + c)*s
in(d*x + c) + a*cos(d*x + c))*sqrt(a^2 - b^2))/(a^2*cos(d*x + c)^2 - 2*a*b*
sin(d*x + c) - a^2 - b^2)) + 12*(6*a^9*b^2 - 7*a^7*b^4 + 2*a^3*b^8 - a*b^10
)*cos(d*x + c) + 6*(2*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*
d*x*cos(d*x + c)^2 - 2*(a^11 - a^9*b^2 - 6*a^7*b^4 + 14*a^5*b^6 - 11*a^3*b^
8 + 3*a*b^10)*d*x + 5*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)*cos(d*x
 + c))*sin(d*x + c))/(3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6
*b^9)*d*cos(d*x + c)^2 - (3*a^14*b - 11*a^12*b^3 + 14*a^10*b^5 - 6*a^8*b^7
- a^6*b^9 + a^4*b^11)*d + ((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^
7*b^8)*d*cos(d*x + c)^2 - (a^15 - a^13*b^2 - 6*a^11*b^4 + 14*a^9*b^6 - 11*a
^7*b^8 + 3*a^5*b^10)*d)*sin(d*x + c)), 1/6*(18*(a^10*b - 4*a^8*b^3 + 6*a^6*
b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*cos(d*x + c)^2 - (36*a^9*b^2 - 68*a^7*b^4 +
43*a^5*b^6 - 11*a^3*b^8)*cos(d*x + c)^3 - 6*(3*a^10*b - 11*a^8*b^3 + 14*a^6
*b^5 - 6*a^4*b^7 - a^2*b^9 + b^11)*d*x - 3*(24*a^8*b^2 - 16*a^6*b^4 + 13*a^
4*b^6 + a^2*b^8 - 2*b^10 - 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8
)*cos(d*x + c)^2 + (8*a^9*b + 16*a^7*b^3 - 17*a^5*b^5 + 19*a^3*b^7 - 6*a*b^
9 - (8*a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^2)*sin(d*x +
 c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(d*x + c) + a)/((a^2 -
 b^2)*cos(d*x + c))) + 6*(6*a^9*b^2 - 7*a^7*b^4 + 2*a^3*b^8 - a*b^10)*cos(d
*x + c) + 3*(2*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x*cos
(d*x + c)^2 - 2*(a^11 - a^9*b^2 - 6*a^7*b^4 + 14*a^5*b^6 - 11*a^3*b^8 + 3*a
*b^10)*d*x + 5*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)*cos(d*x + c))*
sin(d*x + c))/(3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d
*cos(d*x + c)^2 - (3*a^14*b - 11*a^12*b^3 + 14*a^10*b^5 - 6*a^8*b^7 - a^6*b
^9 + a^4*b^11)*d + ((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*
d*cos(d*x + c)^2 - (a^15 - a^13*b^2 - 6*a^11*b^4 + 14*a^9*b^6 - 11*a^7*b^8
+ 3*a^5*b^10)*d)*sin(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\csc\left(c + dx\right)\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))**4,x)
```

```
[Out] Integral((a + b*csc(c + d*x))**(-4), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(228) = 456.
time = 0.45, size = 535, normalized size = 2.24

$$-\frac{\frac{3\,(8\,a^6 b - 8\,a^4 b^3 + 7\,a^2 b^5 - 2\,b^7)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b) + \arctan\left(\frac{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)}{(a^{10} - 3\,a^8 b^2 + 3\,a^6 b^4 - a^4 b^6)\sqrt{-a^2+b^2}} + \frac{18\,a^5 b^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 6\,a^3 b^5 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 3\,a b^7 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 84\,a^6 b^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 12\,a^4 b^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 3\,a^2 b^6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 6\,b^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 104\,a^7 b \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 88\,a^5 b^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 78\,a^3 b^5 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 36\,a b^7 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 228\,a^6 b^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 114\,a^4 b^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 24\,a^2 b^6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 12\,b^8 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 138\,a^5 b^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 96\,a^3 b^5 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 33\,a b^7 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 26\,a^4 b^4 - 17\,a^2 b^6 + 6\,b^8}{(a^9 - 3\,a^7 b^2 + 3\,a^5 b^4 - a^3 b^6)\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + b\right)^3} - \frac{3\,(dx+c)}{a^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c))^4,x, algorithm="giac")

[Out] -1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*d*x + 1/2*c) + a)/sqrt(-a^2 + b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a^2 + b^2)) + (18*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 3*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 84*a^6*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^4*b^4*tan(1/2*d*x + 1/2*c)^4 - 3*a^2*b^6*tan(1/2*d*x + 1/2*c)^4 + 6*b^8*tan(1/2*d*x + 1/2*c)^4 + 104*a^7*b*tan(1/2*d*x + 1/2*c)^3 + 88*a^5*b^3*tan(1/2*d*x + 1/2*c)^3 - 78*a^3*b^5*tan(1/2*d*x + 1/2*c)^3 + 36*a*b^7*tan(1/2*d*x + 1/2*c)^3 + 228*a^6*b^2*tan(1/2*d*x + 1/2*c)^2 - 114*a^4*b^4*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b^6*tan(1/2*d*x + 1/2*c)^2 + 12*b^8*tan(1/2*d*x + 1/2*c)^2 + 138*a^5*b^3*tan(1/2*d*x + 1/2*c) - 96*a^3*b^5*tan(1/2*d*x + 1/2*c) + 33*a*b^7*tan(1/2*d*x + 1/2*c) + 26*a^4*b^4 - 17*a^2*b^6 + 6*b^8)/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(b*tan(1/2*d*x + 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)^3) - 3*(d*x + c)/a^4)/d

Mupad [B]
time = 12.38, size = 2500, normalized size = 10.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/sin(c + d*x))^4,x)

[Out] (2*atan((((8*(4*a^3*b^14 - 24*a^5*b^12 + 60*a^7*b^10 - 80*a^9*b^8 + 60*a^11*b^6 - 24*a^13*b^4 + 4*a^15*b^2))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) - (((8*(4*a^20*b + 2*a^8*b^13 - 14*a^10*b^11 + 30*a^12*b^9 - 30*a^14*b^7 + 20*a^16*b^5 - 12*a^18*b^3))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) - (((8*(4*a^11*b^14 - 24*a^13*b^12 + 60*a^15*b^10 - 80*a^17*b^8 + 60*a^19*b^6 - 24*a^21*b^4 + 4*a^23*b^2))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) + (8*tan(c/2 + (d*x)/2)*(12*a^25*b - 8*a^11*b^15 + 60*a^13*b^13 - 192*a^15*b^11 + 340*a^17*b^9 - 360*a^19*b^7 + 228*a^21*b^5 - 80*a^23*b^3))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2))*1i)/a^4 + (8*tan(c/2 + (d*x)/2)*(8*a^8*b^14 - 52*a^10*b^12 + 140*a^12*b^10 - 220*a^14*b^8 + 220*a^16*b^6 - 128*a^18*b^4 + 32*a^20*b^2))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2))*1i)/a^4 + (8*tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^3*b^15 + 60*a^5*b^13 - 189*a^7*b^11 + 344*a^9*b^9 - 396*a^11*b^7 + 272*a^13*b^5 - 116*a^15*b^3))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2))/a^4 + ((((((8*(4*a^11*b^14 - 24*a^13*b^12 + 60*a^15*b^10 - 80*a^17*b^8 + 60*a^19*b^6 - 24*a^21*b^4 + 4*a^23*b^2))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8

- 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) + (8*tan(c/2 + (d*x)/2)*(12*a^25*
b - 8*a^11*b^15 + 60*a^13*b^13 - 192*a^15*b^11 + 340*a^17*b^9 - 360*a^19*b^
7 + 228*a^21*b^5 - 80*a^23*b^3))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b
^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2))*1i)/a^4 + (8*(4*a^20*b + 2*a^
8*b^13 - 14*a^10*b^11 + 30*a^12*b^9 - 30*a^14*b^7 + 20*a^16*b^5 - 12*a^18*b
^3))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b
^4 - 6*a^18*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^8*b^14 - 52*a^10*b^12 + 140*a
^12*b^10 - 220*a^14*b^8 + 220*a^16*b^6 - 128*a^18*b^4 + 32*a^20*b^2))/(a^21
 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^1
9*b^2))*1i)/a^4 + (8*(4*a^3*b^14 - 24*a^5*b^12 + 60*a^7*b^10 - 80*a^9*b^8 +
 60*a^11*b^6 - 24*a^13*b^4 + 4*a^15*b^2))/(a^20 + a^8*b^12 - 6*a^10*b^10 +
15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) + (8*tan(c/2 + (d*x)/
2)*(8*a^17*b - 8*a^3*b^15 + 60*a^5*b^13 - 189*a^7*b^11 + 344*a^9*b^9 - 396*
a^11*b^7 + 272*a^13*b^5 - 116*a^15*b^3))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 1
5*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2))/a^4)/((16*(2*b^13 - 1
1*a^2*b^11 + 34*a^4*b^9 - 66*a^6*b^7 + 64*a^8*b^5 - 48*a^10*b^3))/(a^20 + a
^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^
2) - (((8*(4*a^3*b^14 - 24*a^5*b^12 + 60*a^7*b^10 - 80*a^9*b^8 + 60*a^11*b^
6 - 24*a^13*b^4 + 4*a^15*b^2))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8
 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) - (((8*(4*a^20*b + 2*a^8*b^13 -
14*a^10*b^11 + 30*a^12*b^9 - 30*a^14*b^7 + 20*a^16*b^5 - 12*a^18*b^3))/(a^2
0 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^
18*b^2) - (((8*(4*a^11*b^14 - 24*a^13*b^12 + 60*a^15*b^10 - 80*a^17*b^8 + 6
0*a^19*b^6 - 24*a^21*b^4 + 4*a^23*b^2))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15
*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) + (8*tan(c/2 + (d*x)/2)
*(12*a^25*b - 8*a^11*b^15 + 60*a^13*b^13 - 192*a^15*b^11 + 340*a^17*b^9 - 3
60*a^19*b^7 + 228*a^21*b^5 - 80*a^23*b^3))/(a^21 + a^9*b^12 - 6*a^11*b^10 +
 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2))*1i)/a^4 + (8*tan(c/
2 + (d*x)/2)*(8*a^8*b^14 - 52*a^10*b^12 + 140*a^12*b^10 - 220*a^14*b^8 + 22
0*a^16*b^6 - 128*a^18*b^4 + 32*a^20*b^2))/(a^21 + a^9*b^12 - 6*a^11*b^10 +
15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2))*1i)/a^4 + (8*tan(c/2
 + (d*x)/2)*(8*a^17*b - 8*a^3*b^15 + 60*a^5*b^13 - 189*a^7*b^11 + 344*a^9*b
^9 - 396*a^11*b^7 + 272*a^13*b^5 - 116*a^15*b^3))/(a^21 + a^9*b^12 - 6*a^11
*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2))*1i)/a^4 + ((
(((((8*(4*a^11*b^14 - 24*a^13*b^12 + 60*a^15*b^10 - 80*a^17*b^8 + 60*a^19*b
^6 - 24*a^21*b^4 + 4*a^23*b^2))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^
8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) + (8*tan(c/2 + (d*x)/2)*(12*a^2
5*b - 8*a^11*b^15 + 60*a^13*b^13 - 192*a^15*b^11 + 340*a^17*b^9 - 360*a^19*
b^7 + 228*a^21*b^5 - 80*a^23*b^3))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13
*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2))*1i)/a^4 + (8*(4*a^20*b + 2*
a^8*b^13 - 14*a^10*b^11 + 30*a^12*b^9 - 30*a^14*b^7 + 20*a^16*b^5 - 12*a^18
*b^3))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16
*b^4 - 6*a^18*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^8*b^14 - 52*a^10*b^12 + 140
*a^12*b^10 - 220*a^14*b^8 + 220*a^16*b^6 - 128*a^18*b^4 + 32*a^20*b^2))/(a^
21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a

```
^19*b^2))*1i)/a^4 + (8*(4*a^3*b^14 - 24*a^5*b^12 + 60*a^7*b^10 - 80*a^9*b^8
 + 60*a^11*b^6 - 24*a^13*b^4 + 4*a^15*b^2))/(a^20 + a^8*b^12 - 6*a^10*b^10
+ 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a...
```

3.52 $\int \frac{1}{3+5\csc(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{x}{12}-\frac{5\text{ArcTan}\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{6d}$$

```
[Out] -1/12*x-5/6*arctan(cos(d*x+c)/(3+sin(d*x+c)))/d
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3868, 2736}

$$-\frac{5\text{ArcTan}\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{6d}-\frac{x}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Csc[c + d*x])^(-1),x]
```

```
[Out] -1/12*x - (5*ArcTan[Cos[c + d*x]/(3 + Sin[c + d*x])])/(6*d)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+5\csc(c+dx)}\,dx &= \frac{x}{3}-\frac{1}{3}\int \frac{1}{1+\frac{3}{5}\sin(c+dx)}\,dx\\ &= -\frac{x}{12}-\frac{5\tan^{-1}\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{6d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 66 vs. 2(31) = 62.
time = 0.05, size = 66, normalized size = 2.13

$$\frac{2(c+dx)-5\text{ArcTan}\left(\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Csc[c + d*x])^(-1),x]
```

```
[Out] (2*(c + d*x) - 5*ArcTan[(2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])])/(6*d)
```

Maple [A]
time = 0.07, size = 34, normalized size = 1.10

method	result	size
derivativedivides	$\frac{-\frac{5\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{6}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}}{d}$	34
default	$\frac{-\frac{5\arctan\left(\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{4}\right)}{6}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}}{d}$	34
risch	$\frac{x}{3}-\frac{5i\ln\left(e^{i(dx+c)}+3i\right)}{12d}+\frac{5i\ln\left(e^{i(dx+c)}+\frac{i}{3}\right)}{12d}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*csc(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-5/6*arctan(5/4*tan(1/2*d*x+1/2*c)+3/4)+2/3*arctan(tan(1/2*d*x+1/2*c))
)
```

Maxima [A]
time = 0.48, size = 49, normalized size = 1.58

$$-\frac{5\arctan\left(\frac{5\sin(dx+c)}{4(\cos(dx+c)+1)}+\frac{3}{4}\right)-4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*csc(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(5*arctan(5/4*sin(d*x + c)/(cos(d*x + c) + 1) + 3/4) - 4*arctan(sin(d*
x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 3.44, size = 33, normalized size = 1.06

$$\frac{4\,dx-5\,\arctan\left(\frac{5\,\sin(dx+c)+3}{4\,\cos(dx+c)}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*csc(d*x+c)),x, algorithm="fricas")
[Out] 1/12*(4*d*x - 5*arctan(1/4*(5*sin(d*x + c) + 3)/cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{5\csc\left(c+dx\right)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*csc(d*x+c)),x)
[Out] Integral(1/(5*csc(c + d*x) + 3), x)
```

Giac [A]
time = 0.43, size = 49, normalized size = 1.58

$$-\frac{dx+c+10\,\arctan\left(-\frac{3\,\cos(dx+c)+\sin(dx+c)+3}{\cos(dx+c)-3\,\sin(dx+c)-9}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*csc(d*x+c)),x, algorithm="giac")
[Out] -1/12*(d*x + c + 10*arctan(-(3*cos(d*x + c) + sin(d*x + c) + 3)/(cos(d*x +
c) - 3*sin(d*x + c) - 9)))/d
```

Mupad [B]
time = 0.25, size = 39, normalized size = 1.26

$$\frac{x}{3}-\frac{5\,\mathrm{atan}\left(\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-15}{24\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+20}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5/sin(c + d*x) + 3),x)
[Out] x/3 - (5*atan((7*tan(c/2 + (d*x)/2) - 15)/(24*tan(c/2 + (d*x)/2) + 20)))/(6
*d)
```

3.53 $\int \frac{1}{5+3\csc(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{x}{5}+\frac{3\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

```
[Out] 1/5*x+3/20*ln(3*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d-3/20*ln(cos(1/2*d*
x+1/2*c)+3*sin(1/2*d*x+1/2*c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3868, 2739, 630, 31}

$$\frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{20d}+\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Csc[c + d*x])^(-1),x]
```

```
[Out] x/5 + (3*Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d) - (3*Log[Cos[(c
 + d*x)/2] + 3*Sin[(c + d*x)/2]])/(20*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
```

```
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{5+3\csc(c+dx)}\,dx &= \frac{x}{5}-\frac{1}{5}\int \frac{1}{1+\frac{5}{3}\sin(c+dx)}\,dx\\ &= \frac{x}{5}-\frac{2\text{Subst}\left(\int \frac{1}{1+\frac{10x}{3}+x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{5d}\\ &= \frac{x}{5}-\frac{3\text{Subst}\left(\int \frac{1}{\frac{1}{3}+x}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{20d}+\frac{3\text{Subst}\left(\int \frac{1}{3+x}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{20d}\\ &= \frac{x}{5}+\frac{3\log\left(3+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(1+3\tan\left(\frac{1}{2}(c+dx)\right)\right)}{20d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 67, normalized size = 0.99

$$\frac{4(c+dx)+3\log\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Csc[c + d*x])^(-1),x]
```

```
[Out] (4*(c + d*x) + 3*Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 3*Log[Cos[(c
+ d*x)/2] + 3*Sin[(c + d*x)/2]])/(20*d)
```

Maple [A]
time = 0.07, size = 48, normalized size = 0.71

method	result	size
norman	$\frac{x}{5}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{20d}-\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{20d}$	41
risch	$\frac{x}{5}-\frac{3\ln\left(-\frac{4}{5}+\frac{3i}{5}+e^{i(dx+c)}\right)}{20d}+\frac{3\ln\left(e^{i(dx+c)}+\frac{4}{5}+\frac{3i}{5}\right)}{20d}$	43
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{20}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{20}}{d}$	48
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3\ln\left(3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{20}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\right)}{20}}{d}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*csc(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/5*arctan(tan(1/2*d*x+1/2*c))-3/20*ln(3*tan(1/2*d*x+1/2*c)+1)+3/20*ln
(tan(1/2*d*x+1/2*c)+3))
```

Maxima [A]
time = 0.48, size = 71, normalized size = 1.04

$$\frac{8\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)-3\log\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+1\right)+3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+3\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*csc(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/20*(8*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 3*log(3*sin(d*x + c)/(cos
(d*x + c) + 1) + 1) + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 3))/d
```

Fricas [A]
time = 3.88, size = 52, normalized size = 0.76

$$\frac{8\,dx+3\log\left(4\cos\left(dx+c\right)+3\sin\left(dx+c\right)+5\right)-3\log\left(-4\cos\left(dx+c\right)+3\sin\left(dx+c\right)+5\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*csc(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*d*x + 3*log(4*cos(d*x + c) + 3*sin(d*x + c) + 5) - 3*log(-4*cos(d*x
 + c) + 3*sin(d*x + c) + 5))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{3\csc\left(c+dx\right)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*csc(d*x+c)),x)
```

```
[Out] Integral(1/(3*csc(c + d*x) + 5), x)
```

Giac [A]
time = 0.43, size = 45, normalized size = 0.66

$$\frac{4\,dx+4\,c-3\log\left(\left|3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\right|\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*csc(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/20*(4*d*x + 4*c - 3*log(abs(3*tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(tan(
1/2*d*x + 1/2*c) + 3)))/d
```

Mupad [B]
time = 0.29, size = 27, normalized size = 0.40

$$\frac{x}{5}-\frac{3\,\mathrm{atanh}\left(\frac{1}{2\,\left(\frac{200\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{27}+\frac{20}{9}\right)}+\frac{41}{40}\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3/sin(c + d*x) + 5),x)
```

```
[Out] x/5 - (3*atanh(1/(2*((200*tan(c/2 + (d*x)/2))/27 + 20/9)) + 41/40))/(10*d)
```

3.54 $\int \csc^3(e+fx)(a+b\csc(e+fx))^m\,dx$

Optimal. Leaf size=274

$$-\frac{\cot(e+fx)(a+b\csc(e+fx))^{1+m}}{bf(2+m)}+\frac{\sqrt{2}\,a(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx)),\frac{b(1-\csc(e+fx))}{a+b}\right)\cot}{b^2f(2+m)\sqrt{1+\csc(e+f}}$$

```
[Out] -cot(f*x+e)*(a+b*csc(f*x+e))^(1+m)/b/f/(2+m)+a*(a+b)*AppellF1(1/2,-1-m,1/2,
3/2,b*(1-csc(f*x+e))/(a+b),1/2-1/2*csc(f*x+e))*cot(f*x+e)*(a+b*csc(f*x+e))^
m*2^(1/2)/b^2/f/(2+m)/(((a+b*csc(f*x+e))/(a+b))^m)/(1+csc(f*x+e))^(1/2)-(a^
2+b^2*(1+m))*AppellF1(1/2,-m,1/2,3/2,b*(1-csc(f*x+e))/(a+b),1/2-1/2*csc(f*x
+e))*cot(f*x+e)*(a+b*csc(f*x+e))^m*2^(1/2)/b^2/f/(2+m)/(((a+b*csc(f*x+e))/(
a+b))^m)/(1+csc(f*x+e))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3925, 4092, 3919, 144, 143}

$$-\frac{\sqrt{2}\,(a^2+b^2(m+1))\cot(e+fx)(a+b\csc(e+fx))^m\left(\frac{a+b\csc(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx)),\frac{b(1-\csc(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\csc(e+fx)+1}}+\frac{\sqrt{2}\,a(a+b)\cot(e+fx)(a+b\csc(e+fx))^m\left(\frac{a+b\csc(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx)),\frac{b(1-\csc(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\csc(e+fx)+1}}-\frac{\cot(e+fx)(a+b\csc(e+fx))^{m+1}}{bf(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Csc[e + f*x])^m,x]
```

```
[Out] -((Cot[e + f*x]*(a + b*Csc[e + f*x])^(1 + m))/(b*f*(2 + m))) + (Sqrt[2]*a*(
a + b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Csc[e + f*x])/2, (b*(1 - Csc[e
+ f*x]))/(a + b)]*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b^2*f*(2 + m)*Sqrt[
1 + Csc[e + f*x]]*((a + b*Csc[e + f*x])/(a + b))^m) - (Sqrt[2]*(a^2 + b^2*(
1 + m))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Csc[e + f*x])/2, (b*(1 - Csc[e + f
*x]))/(a + b)]*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b^2*f*(2 + m)*Sqrt[1 +
 Csc[e + f*x]]*((a + b*Csc[e + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3925

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && !LtQ[m, -1]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(e+fx)(a+b\csc(e+fx))^m\,dx = -\frac{\cot(e+fx)(a+b\csc(e+fx))^{1+m}}{bf(2+m)} + \frac{\int \csc(e+fx)(b(1+m)-

```
[In] integrate(csc(f*x+e)^3*(a+b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\csc{(e + fx)})^m \csc^3{(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*csc(f*x+e))**m,x)
```

```
[Out] Integral((a + b*csc(e + f*x))**m*csc(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{\sin(e+f\,x)}\right)^m}{\sin\left(e + f\,x\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sin(e + f*x))^m/sin(e + f*x)^3,x)
```

```
[Out] int((a + b/sin(e + f*x))^m/sin(e + f*x)^3, x)
```

3.55 $\int \csc^2(e+fx)(a+b\csc(e+fx))^m\,dx$

Optimal. Leaf size=220

$$-\frac{\sqrt{2}\,(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx)),\frac{b(1-\csc(e+fx))}{a+b}\right)\cot(e+fx)(a+b\csc(e+fx))^m\left(\frac{a+b\csc(e}{a+b}\right.}{bf\sqrt{1+\csc(e+fx)}}$$

```
[Out] -(a+b)*AppellF1(1/2,-1-m,1/2,3/2,b*(1-csc(f*x+e))/(a+b),1/2-1/2*csc(f*x+e))
*cot(f*x+e)*(a+b*csc(f*x+e))^m*2^(1/2)/b/f/(((a+b*csc(f*x+e))/(a+b))^m)/(1+
csc(f*x+e))^(1/2)+a*AppellF1(1/2,-m,1/2,3/2,b*(1-csc(f*x+e))/(a+b),1/2-1/2*
csc(f*x+e))*cot(f*x+e)*(a+b*csc(f*x+e))^m*2^(1/2)/b/f/(((a+b*csc(f*x+e))/(a
+b))^m)/(1+csc(f*x+e))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3923, 3919, 144, 143}

$$\frac{\sqrt{2}\,a\cot(e+fx)(a+b\csc(e+fx))^m\left(\frac{a+b\csc(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx)),\frac{b(1-\csc(e+fx))}{a+b}\right)}{bf\sqrt{\csc(e+fx)+1}}-\frac{\sqrt{2}\,(a+b)\cot(e+fx)(a+b\csc(e+fx))^m\left(\frac{a+b\csc(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx)),\frac{b(1-\csc(e+fx))}{a+b}\right)}{bf\sqrt{\csc(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Csc[e + f*x])^m,x]
```

```
[Out] -((Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Csc[e + f*x])/2, (b
*(1 - Csc[e + f*x]))/(a + b)]*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*Sqr
t[1 + Csc[e + f*x]]*((a + b*Csc[e + f*x])/(a + b))^m)) + (Sqrt[2]*a*AppellF
1[1/2, 1/2, -m, 3/2, (1 - Csc[e + f*x])/2, (b*(1 - Csc[e + f*x]))/(a + b)]*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*Sqrt[1 + Csc[e + f*x]]*((a + b*Cs
c[e + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3923

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Dist[-a/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + D
ist[1/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
 b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(e+fx)(a+b\csc(e+fx))^m\,dx = \frac{\int \csc(e+fx)(a+b\csc(e+fx))^{1+m}\,dx}{b} - \frac{a\int \csc(e+fx)(a+b\csc(e+fx))^m\,dx}{b}$$
$$= \frac{\cot(e+fx)\text{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \csc(e+fx)\right)}{bf\sqrt{1-\csc(e+fx)}\sqrt{1+\csc(e+fx)}} - \frac{(a\,c}{}$$
$$= -\frac{\left(a\cot(e+fx)(a+b\csc(e+fx))^m\left(-\frac{a+b\csc(e+fx)}{-a-b}\right)^{-m}\right)\text{Subst}}{bf\sqrt{1-\csc(e+fx)}\sqrt{1+\csc(}}$$
$$= -\frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx)),\frac{b(1-\csc(e+fx))}{a+b}\right.}{bf\sqrt{1+\csc(e+}}$$

Mathematica [F]
time = 2.94, size = 0, normalized size = 0.00

$$\int \csc^2(e+fx)(a+b\csc(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Csc[e + f*x])^m,x]
```

```
[Out] Integrate[Csc[e + f*x]^2*(a + b*Csc[e + f*x])^m, x]
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\csc^{2}\left(f x +e \right)\right) \left(a +b \csc \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*csc(f*x+e))^m,x)
[Out] int(csc(f*x+e)^2*(a+b*csc(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*csc(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*csc(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \csc{\left(e + f x \right)}\right)^{m} \csc^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*csc(f*x+e))**m,x)
[Out] Integral((a + b*csc(e + f*x))**m*csc(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*csc(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\sin(e+f\,x)}\right)^m}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sin(e + f*x))^m/sin(e + f*x)^2,x)
[Out] int((a + b/sin(e + f*x))^m/sin(e + f*x)^2, x)
```

3.56 $\int \csc(e+fx)(a+b\csc(e+fx))^m\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx)),\frac{b(1-\csc(e+fx))}{a+b}\right)\cot(e+fx)(a+b\csc(e+fx))^m\left(\frac{a+b\csc(e+fx)}{a+b}\right)^{-m}}{f\sqrt{1+\csc(e+fx)}}$$

```
[Out] -AppellF1(1/2,-m,1/2,3/2,b*(1-csc(f*x+e))/(a+b),1/2-1/2*csc(f*x+e))*cot(f*x
+e)*(a+b*csc(f*x+e))^m*2^(1/2)/f/(((a+b*csc(f*x+e))/(a+b))^m)/(1+csc(f*x+e)
)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3919, 144, 143}

$$-\frac{\sqrt{2}\cot(e+fx)(a+b\csc(e+fx))^m\left(\frac{a+b\csc(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx)),\frac{b(1-\csc(e+fx))}{a+b}\right)}{f\sqrt{\csc(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Csc[e + f*x])/2, (b*(1 - Csc[e
+ f*x]))/(a + b)]*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*Sqrt[1 + Csc[e +
f*x]]*((a + b*Csc[e + f*x])/(a + b))^m))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int \csc(e+fx)(a+b\csc(e+fx))^m\,dx = \frac{\cot(e+fx)\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \csc(e+fx)\right)}{f\sqrt{1-\csc(e+fx)}\sqrt{1+\csc(e+fx)}}$$
$$= \frac{\left(\cot(e+fx)(a+b\csc(e+fx))^m\left(-\frac{a+b\csc(e+fx)}{-a-b}\right)^{-m}\right)\text{Subst}\left(\int \ldots\right.}{f\sqrt{1-\csc(e+fx)}\sqrt{1+\csc(e+\ldots}}$$
$$= -\frac{\sqrt{2}\,F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\csc(e+fx)),\frac{b(1-\csc(e+fx))}{a+b}\right)\cot(e+f\ldots}{f\sqrt{1+\csc(e+fx)}}$$

Mathematica [**F**]
time = 2.12, size = 0, normalized size = 0.00

$$\int \csc(e+fx)(a+b\csc(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[e + f*x]*(a + b*Csc[e + f*x])^m,x]
[Out] Integrate[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x]
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \csc\left(fx+e\right)\left(a+b\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*csc(f*x+e))^m,x)
[Out] int(csc(f*x+e)*(a+b*csc(f*x+e))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e) + a)^m*csc(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\csc(e + fx))^m \csc(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*csc(f*x+e))**m,x)
```

```
[Out] Integral((a + b*csc(e + f*x))**m*csc(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{\sin(e+f\,x)}\right)^m}{\sin\left(e + f\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sin(e + f*x))^m/sin(e + f*x),x)
```

```
[Out] int((a + b/sin(e + f*x))^m/sin(e + f*x), x)
```

3.57 $\int (a+b\csc(e+fx))^m\,dx$

Optimal. Leaf size=15

$$\text{Int}((a+b\csc(e+fx))^m,x)$$

```
[Out] Unintegrable((a+b*csc(f*x+e))^m,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\csc(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Csc[e + f*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Csc[e + f*x])^m, x]
```

Rubi steps

$$\int (a+b\csc(e+fx))^m\,dx=\int (a+b\csc(e+fx))^m\,dx$$

Mathematica [A]
time = 1.70, size = 0, normalized size = 0.00

$$\int (a+b\csc(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Csc[e + f*x])^m,x]
```

```
[Out] Integrate[(a + b*Csc[e + f*x])^m, x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(a+b\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csc(f*x+e))^m,x)
```

```
[Out] int((a+b*csc(f*x+e))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e) + a)^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e) + a)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \csc (e + fx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))**m,x)
[Out] Integral((a + b*csc(e + f*x))**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e) + a)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \left(a + \frac{b}{\sin (e + f\,x)}\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sin(e + f*x))^m,x)
[Out] int((a + b/sin(e + f*x))^m, x)
```

3.58 $\int (a+b\csc(e+fx))^m \sin(e+fx)\,dx$

Optimal. Leaf size=22

$$\text{Int}((a+b\csc(e+fx))^m \sin(e+fx),x)$$

```
[Out] Unintegrable((a+b*csc(f*x+e))^m*sin(f*x+e),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\csc(e+fx))^m \sin(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Csc[e + f*x])^m*Sin[e + f*x],x]
```

```
[Out] Defer[Int][(a + b*Csc[e + f*x])^m*Sin[e + f*x], x]
```

Rubi steps

$$\int (a+b\csc(e+fx))^m \sin(e+fx)\,dx = \int (a+b\csc(e+fx))^m \sin(e+fx)\,dx$$

Mathematica [A]
time = 8.44, size = 0, normalized size = 0.00

$$\int (a+b\csc(e+fx))^m \sin(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Csc[e + f*x])^m*Sin[e + f*x],x]
```

```
[Out] Integrate[(a + b*Csc[e + f*x])^m*Sin[e + f*x], x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\csc\left(fx+e\right)\right)^{m} \sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csc(f*x+e))^m*sin(f*x+e),x)
[Out] int((a+b*csc(f*x+e))^m*sin(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))^m*sin(f*x+e),x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e) + a)^m*sin(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))^m*sin(f*x+e),x, algorithm="fricas")
[Out] integral((b*csc(f*x + e) + a)^m*sin(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\csc(e + fx))^m \sin(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))**m*sin(f*x+e),x)
[Out] Integral((a + b*csc(e + f*x))**m*sin(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))^m*sin(f*x+e),x, algorithm="giac")
[Out] integrate((b*csc(f*x + e) + a)^m*sin(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \sin(e + f\,x)\,\left(a + \frac{b}{\sin(e + f\,x)}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b/sin(e + f*x))^m,x)
[Out] int(sin(e + f*x)*(a + b/sin(e + f*x))^m, x)
```

3.59 $\int (a+b\csc(e+fx))^m \sin^2(e+fx)\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left((a+b\csc(e+fx))^m\sin^2(e+fx),x\right)$$

```
[Out] Unintegrable((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\csc(e+fx))^m\sin^2(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Csc[e + f*x])^m*Sin[e + f*x]^2,x]
```

```
[Out] Defer[Int][(a + b*Csc[e + f*x])^m*Sin[e + f*x]^2, x]
```

Rubi steps

$$\int (a+b\csc(e+fx))^m\sin^2(e+fx)\,dx=\int (a+b\csc(e+fx))^m\sin^2(e+fx)\,dx$$

Mathematica [A]
time = 6.17, size = 0, normalized size = 0.00

$$\int (a+b\csc(e+fx))^m\sin^2(e+fx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Csc[e + f*x])^m*Sin[e + f*x]^2,x]
```

```
[Out] Integrate[(a + b*Csc[e + f*x])^m*Sin[e + f*x]^2, x]
```

Maple [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \left(a+b\csc\left(fx+e\right)\right)^m\left(\sin^2\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x)
[Out] int((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e) + a)^m*sin(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*csc(f*x + e) + a)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\csc{(e + fx)})^m \sin^2{(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))**m*sin(f*x+e)**2,x)
[Out] Integral((a + b*csc(e + f*x))**m*sin(e + f*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e) + a)^m*sin(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sin{(e + f\,x)}^2 \left(a + \frac{b}{\sin{(e + f\,x)}}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b/sin(e + f*x))^m,x)
[Out] int(sin(e + f*x)^2*(a + b/sin(e + f*x))^m, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```